Ludwig-Maximilians-Universität München
Staatswirtschaftliches Institut
Seminar für Versicherungswissenschaft
Übung Versicherungsmärkte WS 2003/2004

Übungsblatt 4

- **4-1 Pareto efficient risk allocation; Borch-condition**
Think of an economy consisting of two risk averse individuals, a and b. They face two possible states of the world. The probability for the realization of state 1 is π, the probability for state 2 is $(1-\pi)$. In state 1, a's (b's) wealth is w_{1a} (w_{1b}), in state 2 it is w_{2a} (w_{2b}). The society's wealth in the two states of the world is simply defined as the sum of the individuals' wealth ($w_1 = w_{1a} + w_{1b}$ and $w_2 = w_{2a} + w_{2b}$). The two agents can now write a contract, in which they reallocate their wealth in both of the two possible states of the world.

  a) Draw the situation in an Edgeworth-Box and show the set of Pareto-efficient risk allocations in case where there is

  (i) no social risk (i.e., social wealth is the same in both states of the world: $w_{1a} + w_{1b} = w_{2a} + w_{2b}$)

  (ii) social risk (i.e., social wealth differs in the two states of the world, respectively). Analyze the situation if one of the agents is risk neutral!

  b) Write the social planners maximization problem and show that in the optimum the Borch-condition $[\frac{\pi u_a'(1)}{(1-\pi)u_a'(2)} = \frac{\pi u_b'(1)}{(1-\pi)u_b'(2)}]$ holds. (Hint: Think of the definition of Pareto efficiency. It is sufficient to give one agent her outside option and, subject to this, to maximize the other's utility)

- **4-2 Insurance supply, risk-exchange**
The situation is the same as in 4-1. Now we define one of the agents as a risk neutral insurer. Formalize her utility maximization problem. (Assume that the insurer M receives a payment I_1 in state 1 and pays I_2 to the insuree K in state 2.) Analyze this problem and interpret your results. Use a two states of the world diagram to illustrate the form of the contract.

- **4-3 Insurance Supply via risk exchange in repeated games**
Let's, for the moment, return to the basic setting of exercise 4-1. We have two risk averse agents whose income is - for simplicity reasons - perfectly negatively correlated. There are two possible states of the world which occur with equal probability ($\pi = 0,5$). Agent 1's income y is 4 in state a and 16 in state b. For

agent 2 it is just vice versa. The preferences of both agents are given by the utility function $u(y) = \ln y$.

a) Show that a risk sharing agreement following the Borch condition improves the situation for both agents ex-ante. Because of the problem's symmetry it is sufficient to show this for only one of the agents. Assume that they use the Nash Bargaining Solution (i.e. equal split) when they share the surplus from risk sharing.

b) Is this agreement ex-post still mutually beneficial?

c) Now we view a setting where the agents interact repeatedly in the same setting. Is it now possible to design an agreement that is self enforcing?

d) Think about critical properties of the game structure and discuss them briefly.

- **4-4 Economies of scale**
  It is often stated that there are economies of scale in the insurance industry.

  a) Think about a way to model an insurance company's cost function. Especially try to find a way to plausibly describe an insurance company's marginal costs.

  b) Now assume a standard risk neutral utility function of the form $u(w) = aw$. Is it, for this specific utility function, true that there are economies of scale in the insurance business?
  (Economies of scale means that the marginal costs are decreasing.)

  c) Do you think there are economies of scale in the real world insurance industry? (Hint: Think of the firms' attitude towards risk and other than risk costs.)

- **4-5 The Arrow-Lind Theorem**
  The compensating risk premium k is defined as $u(y) = \mathrm{E}u[y + k + \tilde{X}]$ According to the Arrow-Lind Theorem both the premium of each risk avers insurer and the sum of all premia decrease as the number of insurers increases.

  a) Give an intuition for the compensating risk premium, e.g. compare the compensating risk premium with the standard equivalent risk premium.

  b) Suppose that $\mathrm{E}(\tilde{X}) = 0$ and that the risky project is shared among all insurers. Using a Taylor-approximation find an approximative value of k.

  c) Then show that both the individual premium and the sum of all premia decrease as the number of investors increases.

  d) Explain the application of the Arrow-Lind-Theorem for insurance.