<u>Insurance Markets Test A: Answers</u>

1. (a) if a$\neq 0$ and b = 1, total utility is state-dependent but marginal utility is state-independent

   (5 points)

   if $a \geq_{<}$ and $b \neq 1$, both total and marginal utility are state-dependent

   (5 points)

   (<u>no</u> loss of marks if they assume $a > 0$. But they <u>must</u> distinguish between state-dependence of total and marginal utility, $b = 1$ or $b \neq 1$).

   (b) $\max_q (1-\pi)[a + b \cdot ln(y - pq)] + \pi ln(y - L + (1-p)q)$
   FOC: $-\frac{p(1-\pi)b}{y-pq^*} + \frac{(1-p)\pi}{y-L+(1-p)q^*} = 0$
   $(1-p)\pi[y - pq^*] = p(1-\pi)b[y - L + (1-p)q^*]$
   $\rightarrow q^* = \frac{[(1-p)\pi - p(1-\pi)b]y + p(1-\pi)bL}{p(1-p)[\pi + b(1-\pi)]} \otimes$

   Fair premium $\rightarrow p = \pi$

   i. Assume $b = 1$. Then $q^* = \frac{p(1-p)L}{p(1-p)} = L$ full cover, standard case so, $\frac{\partial q^*}{\partial y} = 0$

      Since $u_1'(y) =' bu_2'(y), b = 1$ implies marginal utility *<u>not</u>* state-dependent.

      In case $p = \pi$ but $b \neq 1$, we have
      $\partial q^*/\partial y = \frac{(1-p)\pi - p(1-\pi)b}{p(1-p)[\pi + b(1-\pi)]} = \frac{(1-b)}{b}$

   ii. $b < 1$. $u_1'(y) < u_2'(y)$. So we have $q^* > L$ as figure shows.

      In this case $\partial/\partial y = \frac{(1-b)}{b} > 0$

   iii. $b > 1. u_1'(y) > u_2'(y)$ so we have $q^* < L$ as figure shows

      In this case $\partial q^*/\partial y = \frac{(1-b)}{b} < 0$

   (c) From the cover demand function $\otimes$ we have:
   $q^* = \frac{\pi y}{\alpha p(1-p)} + \frac{\beta p}{\alpha p(1-p)}$
   where $\alpha = \pi + b(1-\pi) > 0$, $\beta = [(1-\pi)b(L-y) - \pi_y] < 0$ since $L < y$. Then,
   $\partial q^*/\partial p = -\frac{\pi y \alpha(1-2p)}{[\alpha p(1-p)]^2} + \frac{1}{[\alpha p(1-p)]^2}\{\alpha p(1-p)\beta - \beta p \alpha(1-2p)\}$

$$= -\frac{\pi y\alpha(1-2p)}{[\alpha p(1-p)]^2} + \frac{\alpha p\beta}{\alpha^2 p^2(1-p)^2}\{1-p-1+2p\}$$

$$= -\frac{\pi y\alpha(1-2p)}{[\alpha p(1-p)]^2} + \frac{\beta}{\alpha(1-p)^2} < 0$$

since $p < 1$ (premium *rate*) and $\beta < 0$. So, the demand for cover falls with the premium rate for *all* values of b.

2. The point about this question is that it is not a straight forward application of the standard case presented in the lecture - that would be too easy. They actually have to work things out, rather than *assure* the answer is the same and just present it in general terms. The point is that the two contracts do not give the same expected income to the buyer, and so the answer has to be properly worked out.

   (a) Contract A: Cover is $q_D = max[0, L-20]$
   So expected cost of cover is $\overline{q}_D = \int_0^{20} max[0, L-20]1/100dL$
   since L is uniformly distributed on [0, 100]. So,
   $\overline{q}_D = \int_0^{20} 0 \cdot 1/100dL + \int_{20}^{100}(\frac{L-20}{100})dL$
   $= [\frac{1/2L^2-20L}{100}]_{20}^{100} = 30 + 2 = 32$
   They could get the same answer just by using
   $\overline{q}_D = 0,2(0) + 0,8[\frac{80-0}{2}] = 32$
   This picture below will explain this (if they ask for an explanation in class)

   With prob. 0,2, $L \leq 20$ and q $= 0$
   With prob. 0,8, L is uniformly distributed between 20 and 100, and so q is uniformly distributed between 0 - 80. Thus, the premium of 40 EUR has a positive loading of 8 EUR, or 25% on the fair premium.
   Contract B:Cover is $\overline{q}_c = \int_0^{100} 0,8L1/100dL = 40$
   or alternatively q is uniformly distributed on [0, 80], thus in this case the premium is fair.

   (b) In fact she'll choose contract B (because the expected cost to the insurer of each contract is not the same, but A is profibable, has positive loading, one cannot simply apply the Schlesinger theorem).

   (c) Income under Contract A is $y_D = 200 - 40 - L + max[0, L-20]$
   So its expected value is $\overline{y}_D = 200 - 40 - E[L] + E[max[0, L-20]]$
   $= 200 - 40 - 50 + 32 = 142EUR$
   Income under Contract B is $y_c = 200 - 40 - L + 0,8L = 160 - 0,2L$
   $\overline{y}_c = 160 - 0,2E[L] = 150EUR$

Thus we cannot apply the Schlesinger theorem to conclude that A is better, since expected value of income is lower. However, we cannot conclude that A is worse than B, because income may be less risky enough to compensate for lower expected value. However, by applying the SOSD argument we can show that B will always be preferred to it. Diagram on next page (in fact 1^{st} order stochastic dominance would do, but SOSD certainly holds, too).

Since the area under the distribution for contract B lies always below that for contract A, B SOSD A (FOSD as well).

3. (a) Since the Edgeworth box is a square, the contract curve lies on the positively sloped diagonal, the allocation $y_{i1} = y_{i2} = 80$ is feasible in each state, $i = 1.2$, and so it is Pareto efficient. (This is good enough. However, also correct is to use the first order condition:
$\frac{\pi_1 u_{11}(y^*_{11}}{\pi_2 u_{12}(y^*_{12}} = \frac{\pi_1 u_{21}(y^*_{21}}{\pi_2 u_{22}(y^*_{22}}$
which for these probabilities and utility functions is
$\frac{0{,}5 r_1 e^{-r_1 y_{11}}}{0{,}5 r_1 e^{-r_1 y_{12}}} = \frac{0{,}5 r_2 e^{-r_2 y_{21}}}{0{,}5 r_2 e^{-r_2 y_{21}}}$
which is obviously satisfied at $y_{11} = y_{12}, y_{21} = y_{22}$, and feasibility then gives (80, 80) (of course this is not the only P-E sol^2).

(b)

(c) This is mutual insurance:
1 agrees to compensate 2 in the event state 1 occurs, so is insuring 2 against state 1.
2 agrees to compensate 1 in the event state 2 occurs, so is insuring 1 against state 2.
Alternatively: 1 pays 2 a premium of 20 EUR, = expected value of loss, for sure, i.e. in each state, and gets back 40 EUR if and only if state 2 occurs.
Or: 2 pays 1 ...

(d) Now have to solve explicitly, using FOC's:
$\frac{e^{-y^*_{11}}}{e^{-y^*_{12}}} = \frac{e^{-2y^*_{21}}}{e^{-2y^*_{12}}} \Rightarrow y^*_{12} - y^*_{11} = 2(y^*_{22} - y^*_{21})$
$y^*_{11} + y^*_{21} = 160 \Rightarrow y^*_{11} = 160 - y^*_{21}$
$y^*_{12} + y^*_{22} = 140 \Rightarrow y^*_{12} = 140 - y^*_{22}$
Solving these 3 equations gives the equation of the contract curve (referred to the y^*_{21}, y^*_{22} - axes)
$y^*_{21} = 20/3 + y^*_{22}$
From the figure, we can then find that the allocation
1: 85,73

2: 75,67
is Pareto efficient.

The pairs: 1 gets (85,73), 2 gets (75,67) are Pareto efficient
Interpretation in terms of insurance is just as before.

(e) In this case we know 2 is risk neutral and the solution will be on 1's certainty line. So, in the social risk case, 1 gets (80, 80) is a Pareto efficient allocation. In the no-social-risk case, (80, 80) is also still a Pareto efficient outcome.
Diagram: take either of the straight lines drawn from the initial endowment point as 2's indifference curve.
Insurance interpretation: just as before, except that 2 now <u>fully</u> insures 1.