Appendix to Lecture 2: Example on the superiority of deductibles

1. Consider the following example of a deductible contract D and two alternatives, the first an arbitrarily constructed insurance contract, A, the second a coinsurance contract, C. There are 10 equally probable loss states with $L \in [0, 9]$. We assume the contract D has deductible$= 3$. The expected value of the cost of cover, q, to the insurer is in each case 2.1, and this is assumed also to be the premium amount P paid by the buyer. The value of α in the coinsurance contract is found from

$$\bar{q} = 0.1 \sum q = 0.1\alpha \sum L = 0.1\alpha 45 = 2.1 \tag{1}$$

giving $\alpha = 0.4667$. The income of the buyer in each case is

$$y = y_0 - P - L + q = 17.9 - L + q \tag{2}$$

it being assumed that the buyer's initial income $y_0 = 20$. Finally, we assume that the buyer's utility function is $u = \ln y$.

L	q_D	q_A	q_C	y_D	y_A	y_C
0	0	0	0	17.9	17.9	17.9
1	0	0	0.4667	16.9	16.9	17.3667
2	0	1	0.9334	15.9	16.9	16.8333
3	0	1	1.4001	14.9	15.9	16.3
4	1	2	1.8668	14.9	15.9	15.7667
5	2	2	2.3335	14.9	14.9	15.2333
6	3	3	2.8002	14.9	14.9	14.7
7	4	3	3.2669	14.9	13.9	14.1667
8	5	4	3.7336	14.9	13.9	13.6333
9	6	5	4.2003	14.9	13.9	13.1

Both the alternatives to the deductible contract seek to improve upon it by giving higher cover in the low-loss, higher income states, but the requirement that the expected value of cover (= premium) be unchanged means that they therefore have to give lower cover in the high-loss, lower income states. Given that the buyer is risk averse, this can never be better for him, as we see by computing the expected utility in each case

$$\bar{u}_D = 0.1 \sum lny_D = 2.739 \tag{3}$$

$$\bar{u}_A = 0.1 \sum lny_A = 2.737 \quad (4)$$

$$\bar{u}_C = 0.1 \sum lny_C = 2.489 \quad (5)$$

2. The figure compares D and A, and is the discrete equivalent of the figure used to illustrate the theorem on the superiority of deductibles given in the lecture. That A is only a little worse than the deductible contract in expected utility terms is due to the fact that it is itself very close to being a deductible contract, and we have not established that a deductible of 3 is in fact optimal in the class of deductible contracts.

Exercise: In the example in the table, find the optimal deductible contract, where the deductible is an integer in [1,8]. What if you were allowed the interval [0,8]? Explain your answer in each case. (Remember, the premium has to be always equal to the expected cost of cover to the insurer).