Insurance demand with state dependent utilities

1. It seems reasonable to believe that for at least some types of losses for which insurance can be bought, the utility of income will depend on whether or not the loss has taken place. Sickness is an obvious example. A formal extension of the model of insurance demand to this case is quite simple. More of a challenge is the interpretation of the results.

2. We again take state 1 as the no-loss state and state 2 as the loss-state, but now denote the utility function in state $i = 1, 2$ as $u_i(y_i)$, with $u_1(y') > u_2(y')$, for all $y' > 0$. Otherwise these are standard Neumann-Morgenstern utility functions. For simplicity we shall always assume that insurance is offered at a fair premium, since this brings out clearly the implications of state contingent utilities. Extension to other cases should be straightforward. Let p therefore denote both the probability of loss and the premium rate. Formulating the insurance problem as one of choosing state contingent incomes (the $y-$model), we have to solve

$$max_{y_1 y_2}(1-p)u_1(y_1) + pu_2(y_2) \quad s.t. \;\; (1-p)y_1 + py_2 = \bar{y} \tag{1}$$

where $\bar{y}$ is the expected value of income. Assuming an interior solution, it is easy to see that the optimum requires

$$u_1'(y_1^*) = u_2'(y_2^*) \tag{2}$$

At a fair premium, the insurance buyer will always want to equalise marginal utilities of income across states. Of course, this implies equality of *incomes* across states if and only if the marginal utility of income is not state dependent, which is something of a special case. More generally, we want to see what this condition of equality of marginal utilities implies for the choice of incomes, and therefore of insurance cover, across states, when the marginal utility of income is also state dependent..

3. A nice way of doing this was developed by P J Cook and D A Graham in "The Demand for Insurance and Protection: the Case of Irreplaceable Commodities" (reprinted in Dionne and Harrington, Foundations...), a classic paper which everyone should read. Thus define the consumer's willingness to pay to be in the good state rather than the bad state as $w(y)$ in

$$u_1(y - w(y)) = u_2(y) \tag{3}$$

The notation emphasises that this willingness to pay may depend on the income level. Figure 1 illustrates this in the utility-income space. In the figure, for any given level of y in state 2, $w(y)$ gives the reduction in this income level required to yield an equal level of utility in state 1. It is just the horizontal difference, in the *leftward* direction, between the two curves. The point is that this is a useful way to describe the relationship between the two curves as y changes. To develop this further, note that (3) is an identity, so differentiating through with respect to y gives

$$u_1^{'}(y - w(y))(1 - w^{'}(y)) = u_2^{'}(y) \tag{4}$$

or

$$w^{'}(y) = 1 - \frac{u_2^{'}(y)}{u_1^{'}(y - w(y))} \tag{5}$$

Thus the way in which the willingness to pay changes as income varies is determined by the slopes of the utility functions at equalised utility values. It seems reasonable to assume $w^{'}(y) \geqslant 0$, for example we would expect your willingness to pay to be healthy rather than sick not to fall with your income. This implies that $u_2^{'}(y) \leqslant u_1^{'}(y - w(y))$ for all y. We shall focus on that case here.

4. It is also useful to look at this in the state contingent income space. In the case where utility is not state dependent, we regard the 45^0 line as the certainty line, because equality of incomes implies equality of utilities. In the state dependent utility case, the 45^0 line still corresponds to certainty of income, but it no longer implies certainty of utility. In fact we know that it implies that utility in state 2 is below that in state 1. In order to determine a locus of points at which utility across states is equal, *i.e.* certain, we know from (3) that we have to subtract $w(y)$ from each income level in state 2, the bad state, to obtain the income level in state 1, the good state, that yields the same utility level. Where $w^{'}(y) > 0$, this implies the curve shown as WW in Figure 2, whereas when $w^{'}(y) = 0$ we have the line WW'.

5. We now come to the results. These are:

(a) If $w^{'}(y_1^*) = 0$, then the insured buys full insurance in the sense that $u_1(y_1^*) = u_2(y_2^*)$. This implies that, since $y_2^* > y_1^*$, he buys more than full coverage of any income loss in state 2.

(b) If $w^{'}(y_1^*) > 0$, the insured buys less than full insurance, in the sense that $u_1(y_1^*) > u_2(y_2^*)$. Coverage of any income loss may be more or less than full.

Proof of (a): $w^{'}(y_1^*) = 0 \Rightarrow u_2^{'}(y_2^*) = u_1^{'}(y_2^* - w(y_2^*))$. The first order condition implies $u_2^{'}(y_2^*) = u_1^{'}(y_1^*)$. So $y_1^* = y_2^* - w(y_2^*)$, implying that the tangency must be on the line WW' in Figure 2.

Proof of (b): $w^{'}(y_1^*) > 0 \Rightarrow u_2^{'}(y_2^*) < u_1^{'}(y_2^* - w(y_2^*))$. The first order condition implies $u_2^{'}(y_2^*) = u_1^{'}(y_1^*)$. So diminishing marginal utility implies $y_2^* - w(y_2^*) < y_1^*$. Thus the tangency must be to the right of the curve WW in Figure 2.

Figure 3 illustrates.