Seminar for Insurance Economics
Wintersemester 2001/02

Insurance Markets: Lecture 1

A. Introduction

1. What is traded?

(i) Obvious answer: a certain payment, the premium, is exchanged for the promise to pay partial or full compensation (cover) for loss resulting from carefully specified events. So premium is exchanged for cover.

(ii) Less obvious answer: state-contingent incomes. Income is reduced in states of the world in which the specified events do not happen, and increased in states in which the events do happen, as compared to the situation without insurance.

2. There are many different insurance markets, corresponding to the types of events concerned. We try to abstract the essential features of an insurance market and incorporate them into a simple model. However, in applications to specific insurance markets it will often be necessary to modify and extend the simple model.

3. It is usual in economics to analyse a market in terms of the determinants of demand, of supply, and of market equilibrium. In the first part of this course we follow that procedure, on the assumption that there are no asymmetries of information between buyers and sellers of insurance. We then take two sorts of information asymmetries into account:

(i) Buyers may be imperfectly informed about the likelihood that sellers will actually meet their claims - honour their promises. This can create a "lemons problem". The solution to this has typically been regulation of insurance markets. We examine the economics of insurance market regulation.

(ii) Sellers may be imperfectly informed about buyers' risk types (the *adverse selection* problem) or about the extent to which they incur costs to reduce the likelihood or size of losses (the *moral hazard* problem). We consider in some depth the models proposed to analyse these problems.

B. The Demand for Insurance

4. This is essentially an area of application of expected utility theory. The individual insurance buyer possesses a von Neumann-Morgenstern utility function defined on incomes in the possible states of the world. It remains

just to define the budget constraint appropriately, and then to generate the implications of the model. We begin with the simplest possible model.

5. **The basic model.**

There are 2 states. Income in the no-loss state is $y_1 = y$, income in the loss state is $y_2 = y - L$. Probability of loss L is π. Thus the expected value of income without insurance is

$$\bar{y} = (1-\pi)y + \pi(y-L) = y - \pi L \tag{1}$$

πL is the expected value of income loss. Expected utility in the absence of insurance is

$$\bar{u}^0 = (1-\pi)u(y) + \pi u(y-L) \tag{2}$$

where the utility function $u(y)$ has $u^{'} > 0$, $u^{''} < 0$. Note that *utility is state-independent.*

The insurer offers cover q at a *premium rate* p, where p is a pure number (as is a probability). The *premium amount* (Euro) is pq. We assume the buyer chooses $q \geqslant 0$ to maximise

$$\bar{u}(q) = (1-\pi)u(y-pq) + \pi u(y-L+(1-p)q) \tag{3}$$

giving the first order (Kuhn-Tucker) condition

$$\bar{u}_q = -p(1-\pi)u^{'}(y-pq^*) + (1-p)\pi u^{'}(y-L+(1-p)q^*) \leqslant 0 \quad q^* \geqslant 0 \quad \bar{u}_q q^* = 0 \tag{4}$$

Then

$$q^* > 0 \Rightarrow \frac{1-p}{p} = \frac{(1-\pi)u^{'}(y-pq^*)}{\pi u^{'}(y-L+(1-p)q^*)} \tag{5}$$

while

$$\frac{1-p}{p} < \frac{(1-\pi)u^{'}(y-pq^*)}{\pi u^{'}(y-L+(1-p)q^*)} \Rightarrow q^* = 0 \tag{6}$$

Assuming $q^* > 0$ it is easy to see that the following must hold:

$$\begin{aligned} p &= \pi \Leftrightarrow q^* = L && (7)\\ p &> \pi \Leftrightarrow q^* < L && (8)\\ p &< \pi \Leftrightarrow q^* > L && (9) \end{aligned}$$

In words:

with a *fair premium* there is *full cover*;

with a *positive loading* there is *partial cover*;
with a *negative loading* there is *more than full cover.*
For example

$$p = \pi \Leftrightarrow u^{'}(y - pq^*) = u^{'}(y - L + (1 - p)q^*) \Leftrightarrow q^* = L \tag{10}$$

The insurer may however only offer the contract $q = L$ at premium $P = pL$.

We then have that

$$q^* > 0 \Leftrightarrow \bar{u}(L) \geqslant \bar{u}^0 \tag{11}$$

6. **Illustration**

We can illustrate the above in a diagram by redefining the model. Let the choice variables in the problem be y_1 and y_2 respectively. The buyer's objective is

$$\max_{y_1 y_2} \bar{u} = (1 - \pi)u(y_1) + \pi u(y_2) \tag{12}$$

We now just have to define the budget constraint. Note that

$$y_1 = y - pq \tag{13}$$
$$y_2 = y - L + (1 - p)q \tag{14}$$

Solving for q in the first equation, substituting into the second and rearranging gives

$$(1 - p)[y - y_1] + p[(y - L) - y_2] = 0 \tag{15}$$

or

$$(1 - p)y_1 + py_2 = y - pL \tag{16}$$

We can interpret this as a budget constraint, with $(1 - p)$ the price of y_1, p the price of y_2, and $y - pL$ as "income", a constant, given p. The point where $y_1 = y$, $y_2 = y - L$ clearly satisfies this constraint. Thus we can draw it as a line with slope $-(1 - p)/p$, passing through the point $(y, y - L)$, as shown in the figure. The interpretation: by choosing $q > 0$, the buyer moves leftward from the initial endowment point $(y, y - L)$, and if there are no constraints on how much cover can be bought, all points on the line, including the certain income, are attainable. The rate of exchange of the state contingent incomes is $-(1 - p)/p$. The demand for cover can be interpreted as the demand for y_2, income in the loss state.

To illustrate the results in 5. on cover, define the *expected value line* by

$$(1-\pi)y_1 + \pi y_2 = \bar{y} \tag{17}$$

This is clearly also a line passing through the initial endowment point $(y, y-L)$, with slope $-(1-\pi)/\pi$. Moreover, we know that any indifference curve, representing a given expected utility in (y_1, y_2)-space, has a slope of $-(1-\pi)/\pi$ at the point at which it cuts the *certainty line OC*. Then clearly the cases of full, partial and more than full cover correspond to the cases in which the budget constraint defined by p is respectively coincident with, flatter than, or steeper than the expected value line (see the figure), since the coverage chosen is always at a point of tangency between an indifference curve and a budget line, for $q^* > 0$. Note that if the budget line is so flat that it does not intersect the indifference curve passing through the initial endowment point, then we have the case where $q^* = 0$, the buyer stays at the initial endowment point. Finally, if only full or zero cover are available, the buyer takes full cover if and only if the resulting point on the certainty line OC is above the *certainty equivalent* of the initial endowment point at $\tilde{y}$.

7. For later purposes it will be useful to be able to read off from the diagram the amount of cover bought. The next figure shows how to do this. Given the optimal point a, draw a line parallel to the certainty line. This cuts the line ce at b. Then the length be represents the coverage bought. To see this note that $ed = pq^*$, while $bd = ad = (1-p)q^*$. So $be = bd + de = pq^* + (1-p)q^* = q^*$.

8. We have above in fact two models in terms of which to discuss the demand for insurance. The q-model allows us to solve for optimal *cover* as a function of the parameters of the problem

$$q^* = q(p, \pi, L, y) \tag{18}$$

The $y-$model allows us to solve for the desired *state contingent incomes* as functions of the parameters of the problem

$$y_s^* = y_s(p, \pi, L, y) \quad s = 1, 2 \tag{19}$$

The two models are of course fully equivalent. The q-model is often easier to handle mathematically. The advantage of the y-model on the other hand is that it allows the obvious similarities with the standard consumer theory to be exploited, especially in the diagrammatic version. We will use whichever seems more suitable for the purpose in hand.

9. **Comparative Statics**

As usual in economic models, we want to explore the relationships between the optimal value of the endogenous variable, the demand for insurance, and the exogenous variables that determine it, p, π, L, y. For an algebraic treatment, the $q-$model is more suitable, for a diagrammatic treatment, the $y-$model is better, again because of the analogies with the standard consumer model (this is not to say that the $q-$model does not have a nice diagrammatic treatment, see the Exercise at the end of this note). Note first that if we assume the premium is always fair, full cover is always bought, and so the comparative statics analysis is trivial. We assume (realistically) that $p > \pi$, and so deal only with the case where $0 < q^* < L$ (the case where $q^* > L$ can also be left as an exercise). Thus the first order condition is

$$\bar{u}_q = -p(1-\pi)u^{'}(y - pq^*) + (1-p)\pi u^{'}(y - L + (1-p)q^*) = 0 \qquad (20)$$

Applying standard methods of comp stats we have that

$$\frac{\partial q^*}{\partial y} = -\frac{\bar{u}_{qy}}{\bar{u}_{qq}} \qquad (21)$$

$$\frac{\partial q^*}{\partial L} = -\frac{\bar{u}_{qL}}{\bar{u}_{qq}} \qquad (22)$$

$$\frac{\partial q^*}{\partial p} = -\frac{\bar{u}_{qp}}{\bar{u}_{qq}} \qquad (23)$$

$$\frac{\partial q^*}{\partial \pi} = -\frac{\bar{u}_{q\pi}}{\bar{u}_{qq}} \qquad (24)$$

Since, because of risk aversion ($u^{''} < 0$) it is easy to show that $\bar{u}_{qq} < 0$, the sign of these derivatives is in each case determined by that of the numerator. Then we have:

$$\bar{u}_{qy} = -p(1-\pi)u^{''}(y - pq^*) + (1-p)\pi u^{''}(y - L + (1-p)q^*) \gtreqless 0 \qquad (25)$$

The indeterminacy of the sign of this effect should not come as a surprise to anyone who knows standard consumer theory: income effects can typically go either way. Thus insurance cover can be an inferior or a normal good. It is however of interest to relate this to the buyer's risk preferences. Thus let

$$y_1^* \equiv y - pq^* \qquad (26)$$

$$y_2^* \equiv y - L + (1-p)q^* \qquad (27)$$

be optimal income in the two states, with $y_1^* > y_2^*$ because of partial cover, and note that from the first order condition we have

$$p(1-\pi) = \frac{(1-p)\pi u^{'}(y_2^*)}{u^{'}(y_1^*)} \tag{28}$$

Substituting gives

$$\begin{aligned} \bar{u}_{qy} &= -u^{''}(y_1^*)\frac{(1-p)\pi u^{'}(y_2^*)}{u^{'}(y_1^*)} + (1-p)\pi u^{''}(y_2^*) && (29) \\ &= (1-p)\pi u^{'}(y_2^*)[\frac{u^{''}(y_2^*)}{u^{'}(y_2^*)} - \frac{u^{''}(y_1^*)}{u^{'}(y_1^*)}] && (30) \end{aligned}$$

Recall now the definition of the Pratt-Arrow measure of (absolute) risk aversion

$$A(y) \equiv -\frac{u^{''}(y)}{u^{'}(y)} \tag{31}$$

We can then write

$$\bar{u}_{qy} = (1-p)\pi u^{'}(y_2^*)[A(y_1^*) - A(y_2^*)] \tag{32}$$

Thus

$$\bar{u}_{qy} \gtreqless 0 \Leftrightarrow A(y_1^*) \gtreqless A(y_2^*) \tag{33}$$

Since $y_1^* > y_2^*$, insurance cover is a normal good if risk aversion increases with income ($A(y_1^*) > A(y_2^*)$), and an inferior good if risk aversion decreases or is constant with income ($A(y_1^*) \leqslant A(y_2^*)$). Since the latter is what we commonly expect, the somewhat unfortunate conclusion is that insurance is an inferior good. The intuition is straightforward: if an increase in income increases one's willingness to bear risk, then one's demand for insurance falls.

Next

$$\bar{u}_{qL} = -(1-p)\pi u^{''}(y_2^*) > 0 \tag{34}$$

Thus, as we would intuitively expect, an increase in loss increases the demand for cover, *other things being equal.*

Thirdly

$$\bar{u}_{qp} = -[(1-\pi)u^{'}(y_1^*) + \pi u^{'}(y_2^*)] + [p(1-\pi)u^{''}(y_1^*) - (1-p)\pi u^{''}(y_2^*)]q^* \tag{35}$$

But notice that the second term is just $-u_{qy}q^*$. In fact we have a standard Slutsky equation, which we can write as

$$\frac{\partial q^*}{\partial p} = -\frac{\bar{u}_{qp}}{\bar{u}_{qq}} = \frac{(1-\pi)u^{'}(y_1^*) + \pi u^{'}(y_2^*)}{\bar{u}_{qq}} + q^* \frac{u_{qy}}{\bar{u}_{qq}} \tag{36}$$

The first term is the substitution effect, and is certainly negative ($\bar{u}_{qq} < 0$), while the second is the income effect and, as we have seen, could be positive or negative (or zero). If insurance is an inferior good this income effect is negative and so the demand for cover certainly falls as the premium rate (price) rises. That is, there is no ambiguity if absolute risk aversion increases (or is constant) with income. On the other hand if insurance is a normal good the income effect is positive and so works against the substitution effect. That is, insurance may be a Giffen good if risk aversion decreases sufficiently with income.

The intuition is also easy to see. A fall in the premium rate reduces the price of income in state 2 relative to that in state 1, and so, with utility held constant, y_2 will be substituted for y_1, implying an increased demand for cover. However, the fall in premium also increases real income, to an extent dependent on the amount of cover already bought, q^*, and this will tend to reduce the demand for insurance if risk aversion falls with income, and increase it if risk aversion increases with income.

Finally we have

$$\bar{u}_{q\pi} = pu^{'}(y_1^*) + (1-p)u^{'}(y_2^*) > 0 \tag{37}$$

Thus, as we would expect, an increase in the risk of loss increases demand for cover. Note, however, there is a strong "other things equal" assumption here. In general we would not expect the premium to remain constant when the loss probability changes, though we need some theory of the supply side of the market before we can predict how it would change. Thus the above does not give the full market comparative statics of a change in loss probability. Exactly the same point applies to the change in L.

We carry out the diagrammatic comparative statics analysis by using the $y-$model and the state contingent income space. This cannot of course add anything to the above results but may help with the intuition.

(Figures will be presented in the lecture)

Exercises.

1. Construct a diagram with **premium amount** on the horizontal axis and **cover** on the vertical axis, on the assumption that the **premium rate** is a given constant, and replicate the diagrammatic analysis, carried out above in the state-contingent income space (equilibrium as well as comparative statics), in this diagram.
2. Analyse the comparative statics in the case in which $p < \pi$.