Seminar for Insurance Economics
Wintersemester 2001/02

Insurance Markets: Lecture 2

Coinsurance and deductibles.

1. The simple two-state model considered in the previous lecture is useful, but of course limited. One aspect of this limitation is that the idea of "partial cover" is very simple: in the single loss-state, $q < L$. In reality, when there are multiple loss states, there can be different types of partial cover. One is the case of **coinsurance**, where a fixed proportion of the loss is paid in each state. The other is the case of a **deductible**, where nothing is paid for losses below a specified value, and when losses exceed this, the insured receives an amount equal to the loss minus the specified value, called the deductible. In practice, the latter is much the more commonly observed form of partial cover. We now examine why this is. In fact it can be shown that, when offered a choice between a contract with a deductible and any other contract with the same premium, assumed to depend only on expected cost to the insurer, a risk averse buyer will always choose the deductible.

2. We generalise the two-state model by assuming now that possible loss lies in some given interval: $L \in [0, L_m]$, and has a given probability function $F(L)$ with density $f(L) = F'(L)$. Thus there is a continuum of possible states of the world and L is a random variable. Under coinsurance we have cover

$$q = \alpha L \quad \alpha \in [0,1] \tag{1}$$

with $\alpha = 0$ implying no insurance and $\alpha = 1$ implying full cover. Under a deductible we have

$$q = 0 \quad L \leqslant D \tag{2}$$

$$q = L - D \quad L > D \tag{3}$$

where D denotes the deductible, with $D = L_m$ implying no insurance and $D = 0$ full cover. The difference between the two contracts is illustrated in Figure 1, which shows cover as a function of loss. Given the premium amount P, and an endowed income y_0 in the absence of loss, the buyer's state-contingent income in the case of coinsurance is

$$y = y_0 - L - P + q = y_0 - (1 - \alpha)L - P \tag{4}$$

and in the case of a deductible is

$$y = y_0 - L - P + q = y_0 - L - P + \max(0, L - D) \tag{5}$$

Figure 2 shows these incomes. The important thing to note about a deductible is that for $L \geqslant D$, the consumer's income becomes certain, and equal to

$$y_D = y_0 - L - P + (L - D) = y_0 - P - D \tag{6}$$

It is this fact that accounts for the superiority, to a risk-averse buyer, of the deductible contract over other forms of contract with the same premium (and expected cost to the insurer). Under a deductible his income cannot fall below y_D, however high the loss.

3. Consider now the probability function for the buyer's income under a given deductible contract. We have

$$\begin{aligned} prob(y &\leqslant y') \;\; for \;\; y' \in [y_D, y_0 - P]) && (7)\\ &= prob(L \geqslant L') \;\; for \;\; L' \in [0, D] && (8)\\ &= 1 - F(L') && (9)\\ prob(y &< y_D) = 0 && (10)\end{aligned}$$

This function $H(y)$ is illustrated in Figure 3. To the left of y_D it is just the horizontal axis, to the right it is determined by $F(L)$. Now consider any other type of contract with the same expected value of cover as the deductible contract in question, and therefore the same premium. We can show that it must have the kind of distribution function shown in the figure as $G(y)$, with the area abc equal to the area cde. But this then means that $H(y)$ is better than $G(y)$ in the sense of Second Order Stochastic Dominance (see note at end), that is, $H(y)$ would be preferred to $G(y)$ by any risk averse buyer. In other words $G(y)$ is riskier than $H(y)$ but has the same expected value.

4. To see that any alternative to the deductible contract must have the general properties of $G(y)$, note the following points:

(i) If the deductible contract and the alternative contract have the same expected cost, *i.e.* value of cover, to the insurer, they imply the same expected income to the buyer. The expected value of income under each of the contracts is $E[y_0 - L - P + q]$, and so, given that $y_0, E[L]$ and P are the same in each case, $E_G[q] = E_H[q]$ implies that the expected incomes are equal.

(ii) It is a standard result that if two distributions with the same support have the same expected value, then the areas under the distribution functions are equal (see note at end). Thus the areas under $H(y)$ and $G(y)$ in the figure must be the same.

(iii) This implies that if a contract tries to improve on the deductible by having a lower distribution function to the right of D in the figure, it must have a higher distribution function to the left of D, and the corresponding areas must be the same.

5. This result was first proved by K J Arrow, but the elegant version given here is due to H Schlesinger. It is very general, and perhaps confirms why deductibles are by far the most common form of partial cover observed in practice.

Note 1: If two distributions on the same support have the same expected value, then the areas under their distribution functions are equal.

Formally, if $G(y)$ and $H(y)$, with $y \in [y_0, y_1]$, satisfy

$$\int_{y_0}^{y_1} y dG(y) = \int_{y_0}^{y_1} y dH(y) \tag{11}$$

then

$$\int_{y_0}^{y_1} G(y) dy = \int_{y_0}^{y_1} H(y) dy \tag{12}$$

Proof: from the first equation we have

$$\int_{y_0}^{y_1} y[G'(y) - H'(y)] dy = 0 \tag{13}$$

Integrating by parts gives

$$\int_{y_0}^{y_1} y[G'(y) - H'(y)] dy = [y(G(y) - H(y))]_{y_0}^{y_1} - \int_{y_0}^{y_1} [G(y) - H(y)] dy = 0 \tag{14}$$

But the first term on the right hand side is zero, since $G(y_0) = H(y_0) = 0$, and $G(y_1) = H(y_1) = 1$, and this gives the result.

Note 2: Definition of Second Order Stochastic Dominance

The distribution $H(y)$ second order stochastically dominates $G(y)$ if

$$\int_{y_0}^{a} H(y) dy \leqslant \int_{y_0}^{a} G(y) dy \tag{15}$$

holds for all $a \in [y_0, y_1]$, with strict inequality for at least some a.