Seminar for Insurance Economics
Wintersemester 2001/02

Insurance Markets: Lecture 7

The Raviv Model

1. The importance of the Raviv model is that it shows how the existence of deductibles and coinsurance in the (optimal) insurance contract is related to the risk aversion of the insurer and the nature of transactions costs. It puts together the demand and supply sides of the insurance market (though in the form of *one* insurance buyer vis à vis *one* insurance seller) to determine the equilibrium insurance contract. The model also shows that less than full coverage can characterise an insurance contract even if there is no adverse selection or moral hazard.

2. Raviv uses the methods of dynamic optimisation to derive the results of the model. This makes his paper hard to follow if you are not already familiar with these methods. However the main results can be established, albeit less rigorously, with a much simpler approach. We set this out here.

3. There is a single insurance buyer and a single insurance seller. The buyer faces a loss $x \in [0, x_m]$. Her income in state x is

$$y(x) = y_0 - P - x + C(x) \tag{1}$$

where P is the premium amount under the insurance contract and $C(x)$ is cover as a function of loss. An important restriction is

$$0 \leqslant C(x) \leqslant x \tag{2}$$

The buyer's utility function is $u(y(x))$, with $u^{'} > 0$, $u^{''} < 0$, she is strictly risk averse. The seller's income is

$$z(x) = P - C(x) - \gamma(C(x)) \tag{3}$$

Here, $\gamma(C(x))$ is the insurance cost function, giving the administrative and transactions cost of the insurance seller as a function of the amount of cover. We assume

$$\gamma(0) = F \geqslant 0, \quad \gamma^{'}(.) \geqslant 0, \gamma^{''}(.) \geqslant 0 \tag{4}$$

That is, there may be a fixed cost, marginal cost may be zero or positive, and, if positive, may be constant or increasing. We assume that the insurer is risk neutral (we know from the earlier analysis that if the insurer is risk averse there will be coinsurance) in order to focus on the effects of transactions costs. Note also that $\partial z/\partial C = -[1 + \gamma^{'}(C(x))]$

4. We assume that the equilibrium contract, consisting of a premium amount P and a coverage function $C(x)$, is Pareto efficient. We therefore find it by solving the problem

$$\begin{aligned} \max_{P, C(x)} \bar{u} &= \int_0^{x_m} u(y(x)) f(x) dx & (5) \\ s.t. \int_0^{x_m} z(x) f(x) dx &= v_0 & (6) \\ 0 &\leqslant C(x) \leqslant x & (7) \end{aligned}$$

Writing the Lagrange function for this problem as

$$\Lambda = \int_0^{x_m} u(y(x)) f(x) dx + \lambda\left(\int_0^{x_m} z(x) f(x) dx - v_0\right) \quad (8)$$

we have the first order (Kuhn-Tucker) conditions

$$\begin{aligned} \frac{\partial \Lambda}{\partial C} &= f(x) u'(y^*(x)) - \lambda^* f(x)[1 + \gamma^{'}(C^*(x))] \leqslant 0 & (9) \\ C^*(x) &\geqslant 0 \;\; C^* \frac{\partial \Lambda}{\partial C} = 0 & (10) \\ \frac{\partial \Lambda}{\partial P} &= -\int_0^{x_m} u^{'}(y^*(x)) f(x) dx + \lambda^* = 0 & (11) \\ \frac{\partial \Lambda}{\partial \lambda} &= \int_0^{x_m} z^*(x) f(x) dx - v_0 = 0 & (12) \end{aligned}$$

We assume that cover is positive for at least some x, so that $P > 0$, otherwise there is nothing to talk about. Note that we have treated the problem as one in the *pointwise* maximisation with respect to C. This can be shown to deliver the correct conditions. Note also we have ignored the upper bound on C, which can also be justified - see the original paper by Raviv.

5. We consider the possibility of the follwing two types of contract (see figure):

(a) **Deductible contract**. Over some interval of losses $[0, D]$ there is zero cover, while over the interval $(D, x_m]$ cover is positive.

(b) **No-deductible contract**. Cover is positive over the entire interval $[0, x_m]$

In each case, it is also of interest to ask about the relationship between loss and cover when cover is positive, is there **full cover above a deductible**($C = x - D$), **full cover** ($C = x$), or **coinsurance above a deductible** ($0 < C < x - D$)? In fact we shall show the following:

(i) There is a deductible contract if and only if $\gamma^{'}(.) > 0$, marginal cost is positive.

(ii) There is full cover if $\gamma^{'}(.) = 0$.

(iii) There is coinsurance above a deductible if and only if $\gamma^{'}(.) > 0$ and $\gamma^{''}(.) > 0$, marginal costs are positive and increasing.

(The first result holds whether or not the insurer is risk averse. The second changes to "coinsurance without deductible" if the insurer is risk averse. In the third case risk aversion of the insurer implies we remove the "only if", there is coinsurance above a deductible regardless of whether or not $\gamma^{''}(.) > 0$).

6. The first main result is

The contract has a deductible if and only if $\gamma^{'}(.) > 0$.

Proof: This is equivalent to saying that the contract has no deductible, i.e. $C^*(x) > 0$ for **all** x, if and only if $\gamma^{'}(.) = 0$.

(a) $\gamma^{'}(.) = 0 \Rightarrow C^*(x) > 0$ for **all** x

We prove this by contradiction. Suppose $\gamma^{'}(.) = 0$ but $C^*(x) = 0$ on some interval, say $[0, D)$. Then we have from the first order condition

$$u'(y^*(x)) < \lambda^* \quad x \in [0, D) \tag{13}$$
$$u'(y^*(x)) = \lambda^* \quad x \in [D, x_m] \tag{14}$$

Multiplying through by $f(x)$ and integrating gives

$$\int_0^D u'(y^*(x))f(x)dx < \lambda^* \int_0^D f(x)dx \quad x \in [0, D) \tag{15}$$
$$\int_D^{x_m} u'(y^*(x))f(x)dx = \lambda^* \int_0^D f(x)dx \quad x \in [D, x_m] \tag{16}$$

Adding, and recalling that $\int_0^{x_m} f(x)dx = 1$ gives

$$\int_0^{x_m} u'(y^*(x))f(x)dx < \lambda^* \tag{17}$$

which contradicts condition (11). Thus $\gamma^{'}(.) = 0$ is a sufficient condition for $C^*(x) > 0$ for all x.

(b) $C^*(x) > 0$ for all $x \Rightarrow \gamma^{'}(.) = 0$

Again we prove this by contradiction. Suppose $C^*(x) > 0$ for all x but $\gamma^{'}(.) > 0$. Then from the first order conditions we have

$$u'(y^*(x)) = \lambda^*[1 + \gamma^{'}(C^*(x))] \tag{18}$$

Multiplying through by $f(x)$ and integrating then gives

$$\int_0^{x_m} u'(y^*(x))f(x)dx = \lambda^* \int_0^{x_m} [1 + \gamma^{'}(C^*(x))]f(x)dx > \lambda^* \tag{19}$$

which again contradicts condition (11). Thus $\gamma^{'}(.) = 0$ is a necessary condition for $C^*(x) > 0$ for all x.

7. We now want to examine the form of the relationship between optimal cover and loss when $\gamma^{'}(.) > 0$, i.e. there is a deductible. Thus we have

$$u'(y_0 - P^* - x + C^*(x)) = \lambda^*[1 + \gamma^{'}(C^*(x))] \quad x \in [D, x_m] \tag{20}$$

This is an identity in x, so differentiating through with respect to x we get

$$-u^{''} + u^{''}\frac{dC^*}{dx} = \lambda^*\gamma^{''}\frac{dC^*}{dx} \tag{21}$$

The first order condition gives $\lambda^* = u'/(1 + \gamma^{'})$, and so substituting and rearranging gives

$$\frac{dC^*}{dx} = \frac{A}{A + \frac{\gamma^{''}}{1+\gamma^{'}}} \tag{22}$$

where $A = -u^{''}/u^{'}$ is the Pratt-Arrow measure of risk aversion for the insurance buyer. This immediately gives the following results:

(a) If $\gamma^{''} = 0$, $\frac{dC^*}{dx} = 1$, and so, given $\gamma' > 0$, we have $C^* = x - D$, full insurance above a deductible.

(b) If $\gamma^{''} > 0$, $\frac{dC^*}{dx} < 1$, and so, given $\gamma' > 0$, we have $C^* < x - D$, coinsurance above a deductible.

8. Note finally that if $\gamma' = 0$, this implies $\gamma^{''} = 0$, in which case we have both no deductible, $D = 0$, and no coinsurance, $\frac{dC^*}{dx} = 1$, i.e. full cover given zero marginal transactions costs and a risk neutral insurer.