November 21, 2001

Seminar for Insurance Economics
Wintersemester 2001/02

Insurance Markets: Lecture 6

The Insurance Firm as Price Taker.

1. We begin with a model that focusses on the choice of a level of reserves by a price taking insurance firm. That is, the insurer takes the prevailing market premium rate as given, and, by offering this rate, receives a given flow of insurance buyers. There is an infinite sequence of time periods of given length. At the beginning of each period the insurer must decide on a level of capital K for the insurance business, in the light of a given distribution of claims costs C, described by the distribution function $F(C)$ with (differentiable) density $f(C)$, defined over the interval $[0, C_u]$. For the moment, we take it that premium income P is also exogenous, and in particular independent of the level of capital chosen (this is clearly not a satisfactory assumption). It is collected at the beginning of the period and invested along with the capital. To begin with we assume that the only capital market asset is a riskless security with gross return $r > 1$. If at the end of the period assets $A \equiv (P + K)r$ are at least enough to meet claims costs C, then the insurer remains in business and receives a continuation value V that is the expected present value of being in the insurance business at the end of the first period. If claims costs turn out to be greater than assets, the insurer pays out his assets and defaults on the remaining claims, losing the right to the continuation value V. Because of limited liability he does not have to pay out to claimants more than A.

2. The insurer can always choose to guarantee solvency by putting in enough capital, since we have assumed that the upper limit C_u on possible

claims is finite. The question of interest is: under what circumstances would the insurer choose to stay solvent, thus making regulation unnecessary, and what are the likely effects of competition on his choices?

3. We assume the insurer is risk neutral and the only cost of capital put into the insurance business is r, the riskless rate of return on the capital market. It follows that he maximises the expected present value of net wealth from the insurance business

$$V_0(K) = \int_0^A (\frac{V}{r} + K + P - \frac{C}{r})dF - K \quad s.t. K \in [0, K_u] \tag{1}$$

where $K_u = (C_u/r) - P$ is the capital required to ensure no default. Now since at the beginning of each period the problem is identical, we have $V = V_0(K)$, and so using this in (1) and rearranging gives

$$V_0(K) = [\int_0^A (K + P - \frac{C}{r})dF - K]/(1 - \frac{F(A)}{r}) \tag{2}$$

So far nothing beyond differentiability has been assumed for the claims distribution F. Empirically however insurance claims distributions typically belong to the class of "increasing failure rate" distributions, with the property that

$$\frac{d}{dC}[\frac{1 - F(C)}{f(C)}] < 0 \tag{3}$$

An important implication of this property is that *only corner solutions to the insurer's wealth maximisation problem are possible*: either he chooses $K = 0$, or $K = K_u$.

> Given the property of the claims distribution in (3), any solution to the insurer's wealth maximisation problem is a corner solution.

Proof. Suppose not, ie there exists a value $K^* \in (0, K_u)$ such that $V(K^*)$ is a maximum. Then $V_0'(K^*) = 0, V_0''(K^*) \leq 0$. Using (2) to evaluate these derivatives gives

$$V_0'(K^*) = [V_0(K^*)f - (1 - F)]/(1 - \frac{F}{r})] = 0 \tag{4}$$

$$V_0''(K^*) = r[V_0(K^*)f' + f)]/(1 - \frac{F}{r})] \leq 0 \tag{5}$$

Then (4) implies

$$V_0(K^*) = (1 - F)/f \tag{6}$$

while (3) implies

$$f^2 + f'(1 - F) > 0 \tag{7}$$

and so substituting for $V_0(K^*)$ from (6) into (5) yields a contradiction.

Note that a solution to the problem does exist, since the objective function is continuous on the compact interval $[0, K_u]$. Which endpoint is optimal is given by the straightforward comparison of the values

$$V_0(0) \quad = \quad F(rP)(P - \frac{\bar{C}_0}{r})r/[r - F(rP)] \tag{8}$$

$$V_0(K_u) \quad = \quad (P - \frac{\bar{C}}{r})r/[r - 1] \tag{9}$$

where $\bar{C}$ is the mean of the claims distribution and $\bar{C}_0 = [F(rP)]^{-1} \int_0^{rP} CdF < \bar{C}$ is the mean of the truncated distribution. As these expressions clearly show, the advantage to not putting up any capital is that the expected present value of claims falls. The disadvantage is that there is a risk of going out of business, $F(rP) < 1$. It does not seem possible to say that one of these endpoints is always better than the other. Figure 1 illustrates the possibilities.

4. There are two major limitations of this model of the insurance firm which could make any policy conclusions derived from it of limited relevance. The first is that the only assets available on the market are safe assets. An interesting question in relation to real insurance companies concerns the interaction between the risks associated with their asset portfolios and those associated with their insurance activities. The second limitation is the exogeneity of the premium income. This is not simply a matter of allowing the firm to choose the premium or volume of insurance sold by maximising profit with respect to a given demand function. More specifically it implies the assumption that *the demand for insurance is independent of the solvency risk of the insurer*. This is a central issue that has to be considered explicitly.

5. We can illustrate the intuition of the above model with a simple example, which shows that given limited liability and an extreme form of asymmetric information it could be in an insurer's interest to run a higher risk of bankruptcy than is desirable from the policyholder's point of view. Consider an individual who faces a 10% chance of a loss of £1,000. The expected value

of loss is £100, but because she is risk-averse assume she is prepared to pay a premium of up to £150 in return for full compensation in the event of loss. A risk-neutral insurer will certainly accept this. If the premium of £150 is paid at the beginning of the period, while the compensation would have to be paid at the end, and the interest rate is 10%, then, to be sure he is able to cover the loss, the insurer will need to put up a starting capital of at least £760, so that the initial investment of £910 will produce £1,000 at the end of the period. Note that putting up this capital is costless to the insurer, since he can invest it as "insurance company capital" at exactly the same rate (in the absence of regulatory restrictions on his portfolio composition) as if he invested it privately on the market. His end-of-period expected wealth is £900, while the expected present value of profit from the insurance business is £150, the premium income, *minus* £100, the expected value of loss, to give a net expected wealth gain of £50.

6. Suppose now that, *unknown to the policy-holder*, the insurer puts up no insurance capital, but instead invests his £760 privately. In the event of loss, he simply pays out £165 at the end of the period and declares the insurance company bankrupt. Then his end-of-period expected wealth is £984.50 (£836 for sure plus $0.9 \times £165$), and the expected present value of profit from the insurance business is the net expected wealth gain of £135. By not putting up any capital the insurer simply *truncates the loss distribution* he faces, thus reducing the expected value of his claims liabilities. In that case his expected wealth gain is £85 higher than if he puts in enough reserves to ensure solvency. The basic point of this simple example can be shown to hold in much more general cases.

7. Two objections can be raised to this example. It may pay the insurer not to put any capital into the insurance business in this one-off case, but what if in fact he is in business "for the long term", *i.e.* the number of periods can be increased indefinitely? If he becomes insolvent, he loses the right to continue in the insurance business in the future, and the loss of future profits may be enough to induce him to put up capital to avoid insolvency today. In this example however this argument does not hold. If in every period the insurer puts the requisite capital into the company his expected present value of profit over an infinite horizon with a 10% interest rate is £550. If he puts up no capital, and allows for the fact that in each period he runs a 10% chance of going out of business, his expected present value of profit is about £660. It is possible to construct more realistic examples, involving continuous loss distributions for which this is not true, and the insurer would find it profitable

to put up the required capital to avoid insolvency. Nevertheless the point remains that there is a wide range of cases in which it pays the insurer to put up none of his own capital.

8. A more fundamental point concerns the buyer's information about the insurer's capital. In the above example, it was assumed that the insurance buyer believed that the insurer would meet her claim, otherwise she would not have bought insurance in the first place - she could have obtained exactly the same degree of coverage in the default case by herself investing the premium. Clearly, if the insurance buyer is fully informed about the default risk, it always pays the insurer to put up the capital, since otherwise he would not be able to sell insurance and would lose even the expected profit of £50. This point can be generalised: if the insurance buyers are fully informed about the risk of insolvency, so that *this is reflected in their willingness to pay* for insurance, then it always pays the insurer to put up enough capital to ensure losses can be met. The intuition is straightforward, and can be given most simply for the case of a risk neutral insurer (the insured is always risk averse). If there is an insolvency risk, the risk averse policy holder would always be prepared to pay more than the fair premium (expected value of loss) to buy insurance against this, and the insurer would always find it profitable to sell it to her. He can only do this however by putting up the required capital (which, recall, is costless to him as long as there are no portfolio restrictions on insurance company investment).

9. **Exercise**

The above took as given the premium and sales of the insurer, and assumed assets were invested risklessly. Now that we have sorted out the issue of capital requirements and solvency, we can build a model that makes endogenous both the premium decision and the choice of possibly risky investment portfolio. Suppose that the insurance market is monopolistically competitive, so that the insurer sees himself as faced with the (inverse) market demand function

$$P = P(n, s) \quad P_n < 0, P_s < 0. \tag{10}$$

where $n \geqslant 0$ is the number of identical insurance contracts sold, P is the premium amount per contract, and $s \in [0, 1]$ is the perceived (by consumers) risk of insolvency. His underwriting net revenue function is then

$$R(n, s) = nP(n, s) - C(n) \quad C' > 0, C'' \geqslant 0 \tag{11}$$

where $C(n)$ is the operating cost function. Underwriting profits are received at the beginning of the period and are immediately invested. Aggregate underwriting losses, paid out at the end of the period, are denoted by L, a random variable with distribution function $F(L,n)$ and density $f(L,n)$. We assume that if $n_1 > n_0$, $F(L,n_0)$ second order stochastically dominates $F(L,n_1)$. In this sense increasing "output" imposes an additional cost on a risk-averse insurer, additional that is to the increasing expected value of aggregate claims as n increases.

The insurer provides a starting capital of K, which has an acquisition cost of $c(K)$, payable at the end of the period, with $c^{'} > 0$, $c^{''} \geqslant 0$. This is meant to include both opportunity cost of capital (therefore non-stochastic) and any other acqusition costs. Also, it must be paid before any claims can be met.

At the beginning of the period, the insurer has available assets for investment of

$$A = R(n,s) + K \tag{12}$$

These assets may be invested in a riskless security with gross return r, and a risky asset which returns $i_1 > 1$ with probability π and $i_2 \leqslant 1$ with probability $1-\pi$. Of course we must have

$$\bar{i} = \pi i_1 + (1-\pi) i_2 > r \tag{13}$$

If the insurer invests the proportion $\theta \in [0,1]$ of his assets A in the risky security, the end of period return on his asset portfolio is

$$W_1 = (\theta i_1 + (1-\theta) r)A = (r + \theta z_1)A \tag{14}$$

with probability π and

$$W_2 = (\theta i_2 + (1-\theta) r)A = (r + \theta z_2)A \tag{15}$$

with probability $(1-\pi)$, where

$$z_j \equiv i_j - r \quad j = 1,2 \tag{16}$$

Then his end of period wealth is the random variable

$$\begin{aligned} Y_j &= W_j - L - c(K) \ \ for\, W_j - c(K) \geqslant L \ \ j = 1,2 && (17) \\ &= 0 \ \ for\, W_j - c(K) < L \ \ j = 1,2 && (18) \end{aligned}$$

and his probability of insolvency is

$$s = \pi \int_{W_1 - c(K)}^{\infty} dF(L) + (1-\pi) \int_{W_2 - c(K)}^{\infty} dF(L) \tag{19}$$

Problem: solve for the insurer's optimal n, K and θ, on the assumption
(a) he is risk neutral
(b) he is risk averse with utility function $u(Y)$.