Seminar for Insurance Economics
Wintersemester 2001/02

Insurance Markets: Lecture 4

Incomplete insurance markets and background risk

1. Up until now, it has been assumed that the insurance buyer faces only one type of loss against which insurance can be bought. In reality insurance markets are typically incomplete, in the sense that not all risks an individual faces can be insured against. Thus one can buy insurance against income loss arising from ill health, but not against income loss due to fluctautions in business conditions leading to loss of overtime, short-time working, and loss of bonuses. In other words part of one's income may be subject to "background risk" which cannot be insured against (or effectively hedged by appropriate choice of share portfolio - note, insurance markets would be redundant if capital markets were complete in the Arrow-Debreu sense). We now want to examine, in the simplest possible model, the effect the existence of an uninsurable risk can have on the purchase of insurance against an insurable risk, as well as the question of whether a welfare loss arises from the absence of a market for insurance against one of the risks. We know that the absence of a market cannot make the insurance buyer better off - one can always choose not to use a market if it is not optimal to do so. The question is whether the consumer is thereby made strictly worse off.

2. Suppose an individual has an income y_0, and faces a loss L with probability π and a loss K with probability θ. There are then four possible states of the world, with associated incomes set out in the following table. It is assumed that insurance cover q can be bought against risk of loss L at premium rate $p \geqslant \pi$. We are interested in the effect of the non-insurability of loss K on the buyer's choice of q.

Loss	0	L
0	$y_1 = y_0 - pq$	$y_3 = y_0 - L + (1-p)q$
K	$y_2 = y_0 - pq - K$	$y_4 = y_0 - L + (1-p)q - K$

The important point to note is that since only L can be insured against, it is possible to use the insurance market to transfer income only between **sets of states,** but not between all individual states**.** Insurance allows income to be exchanged between states 1 and 2, on the one hand, and 3 and 4 on the other, but not between 1 and 2 themselves, or between 3 and 4.

3. Denote the probability of state $s = 1, .., 4$ by ϕ_s. Clearly, since these four states are mutually exclusive and exhaustive, $\sum_s \phi_s = 1$. The exact values of these probabilities ϕ_s will depend on the nature of the stochastic relationship between the two losses. We consider here the three extreme cases:

(i) the two losses are statistically independent. In that case:

$\phi_1 = (1-\pi)(1-\theta)$ - neither loss occurs

$\phi_2 = (1-\pi)\theta$ - only K occurs

$\phi_3 = \pi(1-\theta)$ - only L occurs

$\phi_4 = \pi\theta$ - both losses occur

(ii) the two losses are perfectly positively correlated - either both occur or both do not occur. In effect then, there is only one loss, $L + K$, which for some reason can only be partially insured against. Then

$\phi_1 = (1-\pi) = (1-\theta)$ - neither loss occurs

$\phi_2 = \phi_3 = 0$ - we cannot have only one of the losses occurring

$\phi_4 = \pi = \theta$ - both losses occur

(iii) the losses are perfectly negatively correlated - if one occurs the other does not, and conversely. Then

$\pi = (1-\theta)$, $\theta = (1-\pi)$,

$\phi_1 = \phi_4 = 0$

$\phi_2 = \theta$

$\phi_3 = \pi$

4. The buyer will choose cover to solve

$$\max_q \bar{u}(q) = \sum_{s=1}^{4} \phi_s u(y_s) \quad s.t. \;\; q \geqslant 0 \tag{1}$$

given the specific expressions for y_s in the Table. The general form of the first order condition will be the same for cases (i) - (iii), but the interpretation will of course depend on the precise interpretation of the probabilities ϕ_s, which varies across the three cases. The first order (Kuhn-Tucker condition) is

$$\bar{u}_q = -p[\phi_1 u'(y_1^*) + \phi_2 u'(y_2^*)] + (1-p)[\phi_3 u'(y_3^*) + \phi_4 u'(y_4^*)] \leqslant 0 \tag{2}$$

$$q^* \geqslant 0 \quad \bar{u}_q q^* = 0 \tag{3}$$

It is straightforward to show that the second order condition is satisfied.

The condition shows that if $q^* > 0$,

$$\frac{\phi_1 u'(y_1^*) + \phi_2 u'(y_2^*)}{\phi_3 u'(y_3^*) + \phi_4 u'(y_4^*)} = \frac{(1-p)}{p} \tag{4}$$

Thus the marginal rate of substitution on the left hand side has to be defined with reference to marginal utilities of income averaged over each subset of states within which state contingent incomes can **not** be exchanged. This is simply because an increase in q reduces incomes in **both** states 1 and 2 and increases incomes in **both** states 3 and 4. In order to exchange incomes between states within a subset we would require an insurance market for the loss K. We now want to see what effect the presence of the non-insurable risk has on the purchase of cover against the insurable risk.

5. *Case (i), independence.*

Writing in the explicit expressions for the incomes y_s^* and probabilities ϕ_s we obtain from the first order condition

$$\frac{\pi(1-p)}{p(1-\pi)} \leqslant \frac{(1-\theta)u'(y_0 - pq^*) + \theta u'(y_0 - pq^* - K)}{(1-\theta)u'(y_0 - L + (1-p)q^*) + \theta u'(y_0 - L + (1-p)q^* - K)} \tag{5}$$

$$q^* \geqslant 0 \quad \bar{u}_q q^* = 0 \tag{6}$$

We now have to distinguish between two subcases:

(a) Fair premium, $p = \pi$. Then it is easy to see that $q^* = L$, we have full cover. Thus the background risk makes no difference to the optimal cover against L. To see this, note that the left hand side of the condition becomes 1 in this case. If $q^* < L$, the denominator in the right hand ratio must (because $u'' < 0$) be greater than the numerator, thus the ratio must be < 1 and the condition cannot be satisfied. If however $q^* = L > 0$ the ratio on the right hand side is 1 and equals the left hand side. If $q^* > L$, the numerator on the right hand side is larger than the denominator and the condition is not satisfied. Intuitively, one might think that, when insurance against L is available at a fair premium, one might over-insure, to compensate for not being able to insure against K. In the independence case this intuition is false, because it simply results in expected marginal utility across the states in which L does occur becoming smaller than that across the states in which L does not occur.

(b) Positive loading, $p > \pi$. In that case the ratio on the left hand side becomes $\rho < 1$. Then in that case $q^* = L$ cannot be optimal, because we just saw that the right hand ratio would then equal 1. Assume that $L > q^* > 0$, *i.e.* the loading is not so large that no cover is bought. We want to know what effect on choice of cover introduction of the risk K makes. In general, the answer depends on the precise form of the buyer's utility function. In fact we can show the following:

Introducing K, suitably small, increases cover, if and only if absolute risk aversion decreases with income;

Introducing K, suitably small, reduces cover, if and only if absolute risk aversion increases with income;

Introducing K, suitably small, leaves cover unchanged, if and only if absolute risk aversion is constant.

Proof: We prove only the first, the others follow similarly. Note first that if we want to *increase* the ratio on the right hand side of (5), we have to *increase* q^*, since this *reduces* both incomes and *increases* both marginal utilities in the numerator, and *increases* both incomes and *reduces* both marginal utilities in the denominator.

Now consider the equilibrium in the absence of the risk K. This has to satisfy the condition

$$\rho = \frac{u'(y_0 - pq^*)}{u'(y_0 - L + (1-p)q^*)} \tag{7}$$

We know then, that when we introduce K, since this leaves ρ unchanged, if this reduces the value of the ratio on the right hand side, we will have to increase q^* to restore equality. It is easy to show that the value of the ratio will be reduced (and cover therefore increased) if

$$\frac{u'(y_0 - pq^*)}{u'(y_0 - L + (1-p)q^*)} > \frac{u'(y_0 - pq^* - K)}{u'(y_0 - L + (1-p)q^* - K)} \tag{8}$$

that is, if

$$\frac{u'(y_0 - L + (1-p)q^* - K)}{u'(y_0 - L + (1-p)q^*)} > \frac{u'(y_0 - pq^* - K)}{u'(y_0 - pq^*)} \tag{9}$$

For short, write this as

$$\frac{u'(y_3^* - K)}{u'(y_3^*)} > \frac{u'(y_1^* - K)}{u'(y_1^*)} \tag{10}$$

Now assume that K is sufficiently small that it is permissible to use the simple Taylor series approximations

$$u'(y_s^* - K) \approx u'(y_s^*) - u''(y_s^*)K \quad s = 1, 3 \tag{11}$$

Inserting these into (10) and cancelling terms then gives

$$A(y_3^*) \equiv -\frac{u^{''}(y_3^*)}{u^{'}(y_3^*)} > -\frac{u^{''}(y_1^*)}{u^{'}(y_1^*)} \equiv A(y_1^*) \tag{12}$$

Since $y_3^* < y_1^*$(partial cover), this gives the result.

6. Case (ii): *perfect positive correlation.*

In this case we can show that ideally, if there is fair insurance the buyer would like to set $q^* = L+K$, i.e. over-insure on the $L-$market to compensate for not being able to insure against K. There is no reason why insurers should not sell this insurance because they still break even at the fair premium. If $p > \pi$, the buyer would like to set $q^* < L + K$, for reasons with which we are already familiar, and so we just consider the case of a fair premium. Introducing the appropriate probabilities and incomes for this case into the first order condition gives

$$\frac{(1-\pi)u^{'}(y_0 - pq^*)}{\pi u^{'}(y_0 - L + (1-p)q^* - K)} = \frac{1-p}{p} \tag{13}$$

implying

$$\frac{u^{'}(y_0 - pq^*)}{u^{'}(y_0 - L + (1-p)q^* - K)} = 1 \tag{14}$$

(Note, we can rule out the case in which $q^* = 0$ because then the ratio on the left hand side is strictly less than one, which does not satisfy the Kuhn-Tucker condition). Clearly then this condition is satisfied if and only if $q^* = L + K$. This is then a case in which the noninsurability of K does not reduce welfare, though it does change behaviour. However if, for some reason, cover is restricted in the $L-$market, for example by $q \leqslant L$, then the buyer chooses $q^* = L$ and is made strictly worse off by the non existence of the $K-$market.

7. Case (iii): *perfect negative correlation.*

Inserting the appropriate probabilities and incomes into the first order condition gives

$$\frac{(1-\pi)u^{'}(y_0 - pq^* - K)}{\pi u^{'}(y_0 - L + (1-p)q^*)} \geqslant \frac{1-p}{p} \tag{15}$$

We take the fair premium case, in which the condition becomes

$$u^{'}(y_0 - pq^* - K) \geqslant u^{'}(y_0 - L + (1-p)q^*) \tag{16}$$

Suppose first that $q^* > 0$, so the condition must hold with equality. This then implies

$$pq^* + K = L - (1-p)q^* \tag{17}$$

or

$$q^* = L - K \tag{18}$$

Now L and K are exogenous with $L \gtreqless K$. Thus we have three possibilities:

(a) $L = K$. This implies $q^* = 0$, which is a contradiction. In fact in this case no cover is bought. The reason is that, because of the perfect negative correlation and the equality of K and L, income is certain with zero insurance cover.

(b) $L > K$. Then $q^* = L - K > 0$. In order to equalise incomes across the states, cover has to be bought which just makes up the difference between L and K.

Note a feature of these two cases: the introduction of the second risk K certainly makes a difference to the insurance decision on the purchase of cover on the market for insurance against L, but, because of the perfect negative correlation, there is no welfare loss arising from the absence of a market for insurance against K.

(c) $K > L$. Then we would have $q^* < 0$, which is assumed not to be possible, and again contradicts the assumption that $q^* > 0$. In fact in this case we have $q^* = 0$: buying positive cover would worsen the income inequality between the two states, since it reduces income in the state in which K occurs and L does not. The buyer would actually like to have negative cover, i.e. offer a bet on the occurrence of the loss L, since this would transfer income from the state in which L occurs to that in which K occurs. In this case also, the insurance decision on the $L-$market is certainly affected by the existence of the non insurable risk K. The buyer would be made better off if the $K-$market existed and the $L-$market did not.

Exercise.

1. Find the optimal cover q_L and q_K, given that it can be obtained on competitive insurance markets at premium rates p_L and p_K respectively, first in general, and then for each of the special cases (i) to (iii). Consider both the case of fair insurance and that of a positive loading. Use your results to summarise the cases in which absence of one of the markets makes the buyer strictly worse off.