11. Incomplete markets: background risk

Consider a farmer who faces with probability π an insurable risk to suffer a loss L from a certain wheat disease. In addition he is confronted with the uninsurable risk of losing D with probability τ because of hurricanes, which regularly severly damage the whole country. The four possible states of the world, i. e. no loss at all, a loss of D, a loss of L, and a loss of $L + D$, occur with probabilities f_i, $i \in \{1, 2, 3, 4\}$. Insurance against L is available at rate p.

a) What is the farmer's income y_i in the four possible states of the world? How do π and τ depend on f_i? Write down the farmer's maximization problem with respect to cover against L, s. t. $y_i \geq 0$, and derive the first-order condition (Kuhn-Tucker!). Can the situation with a missing insurance market make the individual better off compared to a full set of insurance markets with fair premia?

b) Suppose that L and D are perfectly negatively correlated, but $L > D$. The premium rate for insurance purchase is now $p = \pi$. Show that the perfect negative correlation provides a partial hedge against the larger insurable loss.

c) Consider the effect of a "small" uninsurable risk on the demand for insurance against the other risk: Suppose that initially $D = 0$, and that there is a premium rate $p \geq \pi$ for cover against L. Assume that initially, even if the premium rate is unfair, the individual buys some positive amount of cover. (What does this mean for the FOC derived above?) Show that the effect of introducing a small uninsurable risk on the demand for insurance against the insurable risk depends on the risk aversion of the insured and the correlation of the insurable and uninsurable risks. What is your answer if the initial premium rate for cover against L is actuarially fair?
(Hint: Use the FOC and its partial derivative with respect to D at $D = 0$ for your analysis.)

12. State dependent utility functions

We usually assume that individuals only suffer monetary losses, or at least that a monetary compensation for loss is possible. In real life, this often seems to be a rather heroic assumption. Being in hospital with a broken leg and having received a fair monetary compensation may not be as good as going skiing. Therefore, we will now consider state dependent utility functions, which capture the idea that individuals may experience and value things differently when in different states of the world.

a) Consider an individual with initial endowment W, probability π for loss L in state 2 and utility functions $u_1(y)$ and $u_2(y)$ for states 1 and 2 respectively. Suppose the individual can buy fair insurance for a premium rate $p = \pi$ and assume that for any income level she derives a higher marginal utility from this income in state 1 than in state 2, i.e. $u_1'(y) > u_2'(y)$. Calculate the slope of the individual's expected utility indifference curve where it crosses the security line. Compare this slope to the slope of the insurance line. What does this mean for the optimal demand for insurance? Draw your result in a two-states-of-the-world-diagram.

b) Now, suppose that the individual's utility functions are state dependent with $u_2(y) = a + bu_1(y)$, with $a < 0$ and $b > 0$. Insurance cover is available at a rate p. What is the effect on the demand for insurance of increases in a and b? Under what circumstances will full cover be bought even though $p > \pi$?

13. Adverse selection in insurance markets

In a particular population everyone runs the risk of loosing \$ 1,000 randomly. Each person's loss occurs independently from anybody else's. The probability π that the loss L occurs depends on the individual's type. 90% of the population are of the l type, whose loss probability π_l equals 10%. The rest of the population is of the h type and faces L with $\pi_h = 60\%$. Every individual knows its type, but nobody else does and there is no way to signal one's type. Each individual's utility is given by $u(y) = 1 - e^{-\lambda y}$. (For this form you have to regard y as a random variable that either equals $y_1 = -pC$ or $y_2 = (1-p)C - L$, depending on the occurance of the loss.)

a) The government regulates the insurance market and only allows pooling contracts. Assume that the government either allows only the same contract to be offered by every company or that there is only one single company in the market. It demands the insurance companies to break even, i.e. to make zero profits.

i) Do there exist pooling full insurance equilibria for $\lambda = 0,002(0,0005)$?

ii) For $\lambda = 0,0005$, does there exist any zero-profit pooling contract which would represent an equilibrium?

b) Now the government abandons regulation, and a competitive insurance market emerges.

i) What happens to a company that still offers a pooling contract?

ii) What are the Rothschild-Stiglitz contracts in this competitive insurance market? (Calculate P_l, P_h and C_h^{RS}, where $P_l = p_l \cdot C_l$. Do not try to calculate C_l^{RS}, but express C_l^{RS} as an implicit function of λ.)