Insurance Markets - Demand for Insurance 2
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The y–Model

Let the choice variables in the problem be y_1 and y_2 respectively. The buyer's objective is

$$\max_{y_1 y_2} \bar{u} = (1-\pi)u(y_1) + \pi u(y_2).$$

Define the budget constraint:

$$\begin{aligned} y_1 &= y - pq && (1) \\ y_2 &= y - L + (1-p)q && (2) \end{aligned}$$

Solving for q in the first equation, substituting into the second and rearranging gives

$$(1-p)y_1 + py_2 = y - pL.$$

Budget constraint:

- $(1-p)$ price of y_1 p the price of y_2
- $y - pL$ "income", a constant, given p

Interpretation:

- by choosing $q > 0$ the buyer moves from the initial endowment point $(y, y - L)$
- if there are no constraints on how much cover can be bought, all points satisfying the constraint, including the certain income, are attainable
- the rate of exchange of the state contingent incomes is $-(1-p)/p$
- the demand for cover can be interpreted as the demand for y_2, income in the loss state

Diagrammatical Interpretation

- The endowment point $y_1 = y$, $y_2 = y - L$ satisfies the budget constraint.
- We can draw the constraint as a line with slope $-(1-p)/p$, passing through the point $(y, y - L)$.
- Define the *expected value line* by

$$(1-\pi)y_1 + \pi y_2 = \bar{y}.$$

- This is also a line passing through the initial endowment point $(y, y - L)$, with slope $-(1-\pi)/\pi$.
- Any indifference curve, representing a given expected utility in (y_1, y_2) -space, has a slope of $-(1-\pi)/\pi$ at the point at which it cuts the *certainty line*.

- The coverage chosen is always at a point of tangency between an indifference curve and a budget line, for $q^* > 0$.

  Thus the cases of **full**, **partial** and **more than full cover** correspond to the cases in which the budget constraint defined by p is respectively **coincident** with, **flatter** than, or **steeper** than the expected value line.

- **Note:** If the budget line is so flat that it does not intersect the indifference curve passing through the initial endowment point, then we have $q^* = 0$.

- **Note:** If only full or zero cover are available, the buyer takes full cover if and only if the resulting point on the certainty line is above the *certainty equivalent* of the initial endowment point at $\tilde{y}$.

We have in fact two models to discuss the demand for insurance.

The q**-model** derives optimal *cover* as a function of the parameters of the problem

$$q^* = q(p, \pi, L, y)$$

The $y-$**model** derives the desired *state contingent incomes* as functions of the parameters of the problem

$$y_s^* = y_s(p, \pi, L, y) \quad s = 1, 2$$

$q-$**model** more suitable for an algebraic treatment

$y-$**model** more suitable for an diagrammatic treatment

Multiple Loss States

- straightforward to extend the model to multiple loss states / continuous losses
- cover as function of loss
- deductible vs. coinsurance $\Rightarrow$ Schlesinger Theorem
  will be treated later on in the course

Incomplete Markets

- individual may be exposed to several risks
- some of them may not be insurable (e.g. income fluctuations due to the business cycle, ...)
- these uninsurable risks affect demand for cover against insurable losses
- effect on demand depends on correlation of risks assume fair premium (otherwise dependent on C/D/IARA)

- **positive correlation**

  demand for cover increased – overinsure in order to insure against non–insurable risk

- **independence**

  demand for cover unchanged

- **negative correlation**

  demand for cover decreased – the two risks help to smooth income without insurance

State Dependent Utility

It seems reasonable to believe that for at least some types of losses for which insurance can be bought, the utility of income will depend on whether or not a particular event takes place, where this event may or may not also cause an income loss.

Obvious example: Sickness – utility of income if one is sick may well differ from that if one is healthy

- state 1 is the no-loss state and state 2 is the loss-state
- denote the utility function in state $s = 1, 2$ as $u_s(y)$
- utilities may differ in **absolute terms**, $u_1(y) > u_2(y) \quad \forall \quad y > 0$ or in **marginal terms**, $u_1^{'}(y) > u_2^{'}(y) \quad \forall \quad y > 0$.

linear example

$u_1(y) = a + bu_2(y) \quad \text{with} \quad a > 0 \quad \text{and} b > 1$

Note that the assumptions on a and b are unnecessarily restrictive.

- otherwise these are standard von Neumann-Morgenstern utility functions
- for simplicity assume insurance is offered at a fair premium p therefore denotes both the probability of loss and the premium rate
- using the y-model, we have to solve

$$max_{y_1 y_2}(1-p)u_1(y_1) + pu_2(y_2) \quad \text{s.t.} \quad (1-p)y_1 + py_2 = \bar{y}$$

where $\bar{y}$ is the expected value of income

- Assuming an interior solution, it is easy to see that the optimum requires

$$u_1'(y_1^*) = u_2'(y_2^*)$$

- At a fair premium, the insurance buyer will always want to equalize marginal utilities of income across states.
- This implies equality of *incomes* across states if and only if the marginal utility of income is not state dependent.
- More generally, we want to see what this condition of equality of marginal utilities implies for the choice of incomes, and therefore of insurance cover, across states, when the utility of income is state dependent.
- We can distinguish three senses in which we could talk of "full insurance":
  - choice of cover that equalizes marginal utilities of income across

states

- choice of cover that equalizes total utilities of income across states
- choice of cover that equalizes income across states.

- When utility is state independent and the premium is fair, these three coincide: choice of cover equalises incomes, marginal and total utilities.
- Interesting implication: An insurance contract that restricts cover to the loss actually incurred - actual loss on income from employment, actual medical costs, in the case of health insurance - is optimal only if marginal utility of income is state independent.