

# Adverse Selection

## so far

- one type of risk (monetary loss  $L$ )
- homogenous risk averse buyers; identical von Neumann – Morgenstern utility function [ $u'(\cdot) > 0, u''(\cdot) < 0$ ]; identical initial wealth  $w$
- risk neutral insurers in a perfectly competitive insurance market

$\Rightarrow$  full insurance with fair premia,

i.e. cover  $C$  equals  $L$  and the premium equals the expected loss

## Now one change

Consumers are heterogeneous in one respect.

There are 2 risk types:

- low risk types with loss probability  $\pi_l$
- high risk types with loss probability  $\pi_h$

where  $\pi_l < \pi_h$  holds

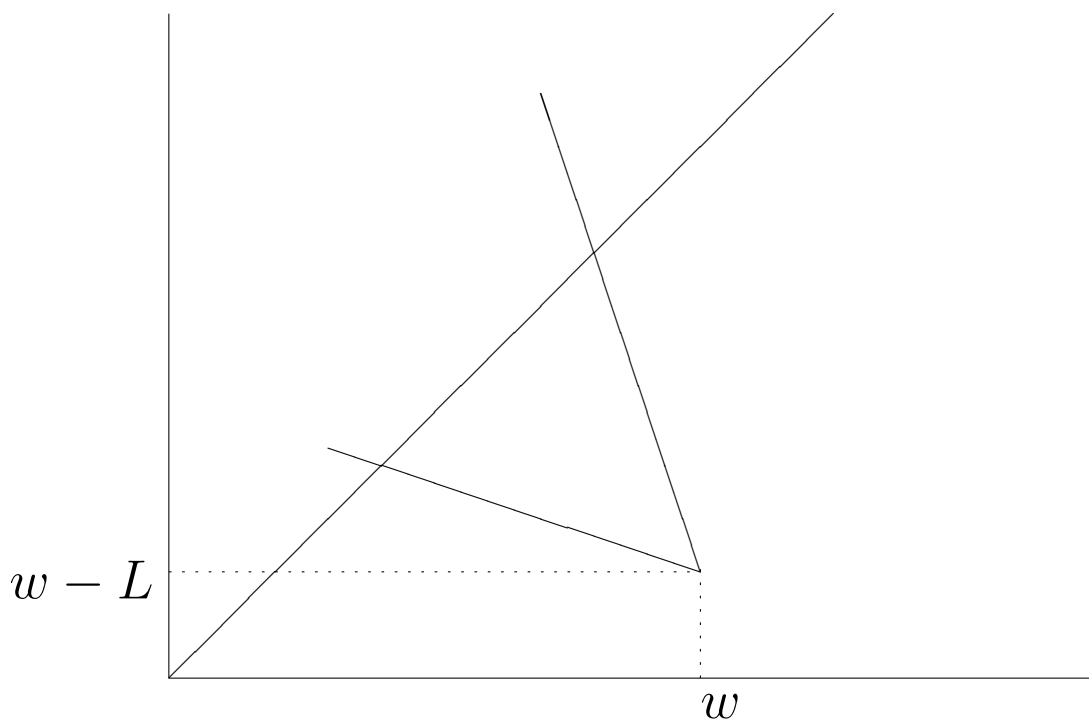
- $\lambda$  is the share of low risk types in the population
- $\bar{\pi} = \lambda\pi_l + (1-\lambda)\pi_h$  is the “pooled loss probability”

One of the problems in class will deal with the case where there are more than 2 types.

## Market equilibrium under sym. information

- Firms make zero profits.
- The contracts maximize buyers' expected utility.

⇒ There exists no other contract that would break even in expectation and would be preferred by at least one consumer.



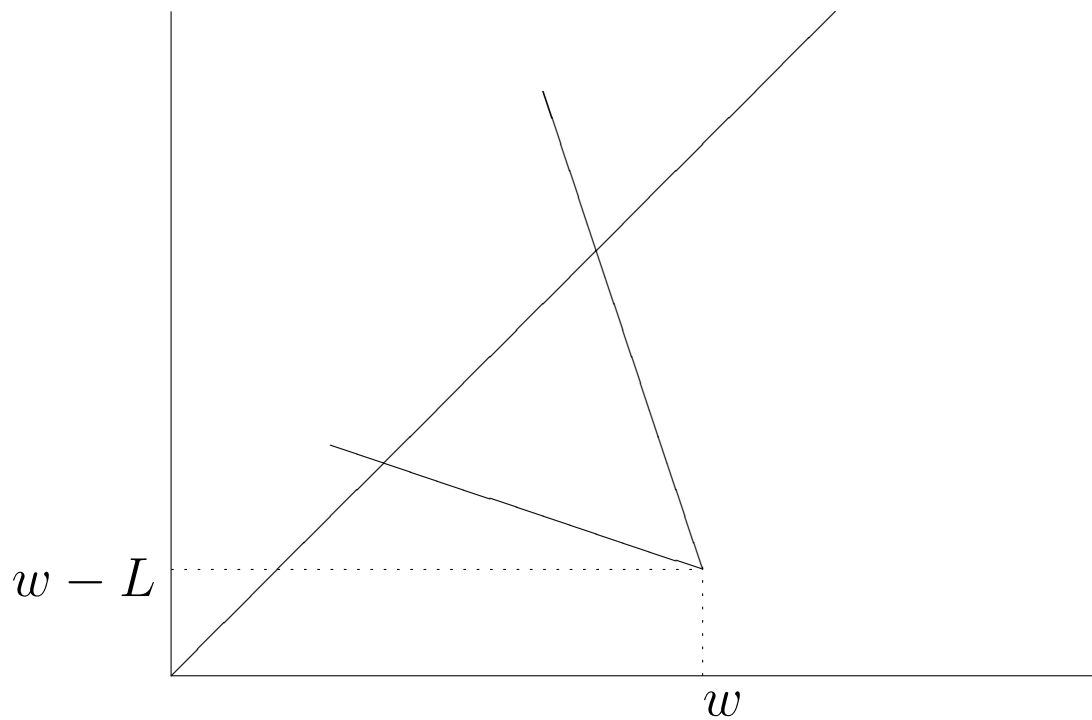
## Asymmetric information

- Buyers know their risk type
- Insurers only know the distribution of risk types in the population

Examples:

- In health insurance buyers know more about their health status.
- In car insurance buyers know more about their driving abilities.

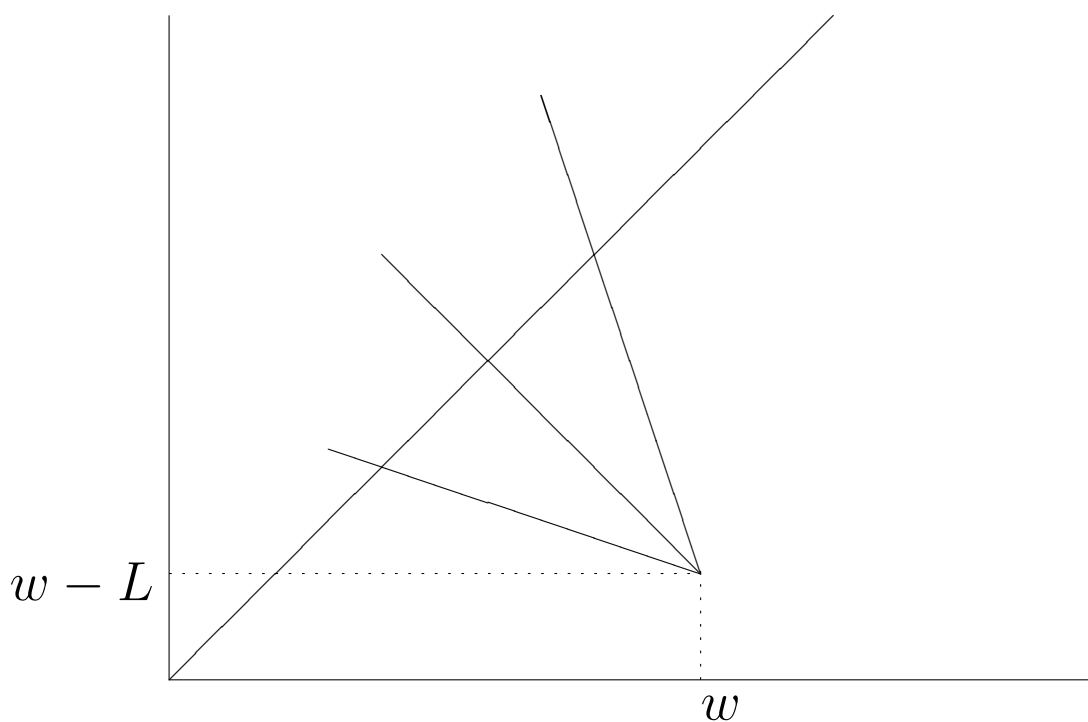
What happens if insurers continue to offer the contracts that were optimal under symmetric information?



⇒ This is the so called phenomenon of “Adverse Selection” (cf. Akerlof (1970): The Market for Lemons)

What else could they do?

What about offering a contract for the actuarially fair pooling premium?



$\Rightarrow$  again: Adverse Selection

What else could be done?

Contracts could specify not only the premium rate but also the cover

We need additional assumptions:

- **Exclusiveness** Every buyer can only contract with one firm and can only buy one contract.
- **Pure Strategy Equilibria** We only look at equilibria in pure strategies and disregard mixed strategies.
- **Symmetric Equilibria** We impose symmetry, i.e. buyers of the same risk type buy the same contracts and firms offer the same menus of contracts.
- **Single Crossing Property** The indifference curves of a high and a low risk buyer only intersect once. This follows from the fact that  $h$  and  $l$  types only differ in their loss probability (cf. problem 6–1)

## 5 steps to Rothschild-Stiglitz

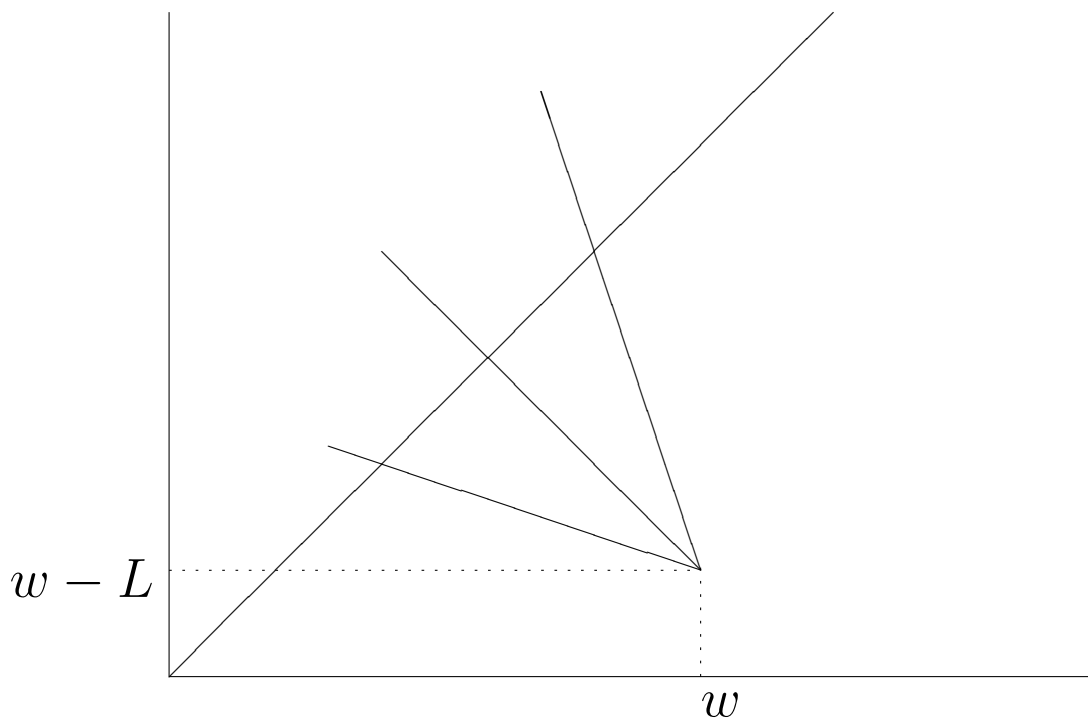
- 1 There exists no pooling equilibrium.
- 2 No contract yields profits in equilibrium.
- 3 No loss making contracts in equilibrium.
- 4 High risk types get full coverage
- 5 Low risk types get partial insurance for an actuarially fair premium



## Step 1 – No pooling equilibrium

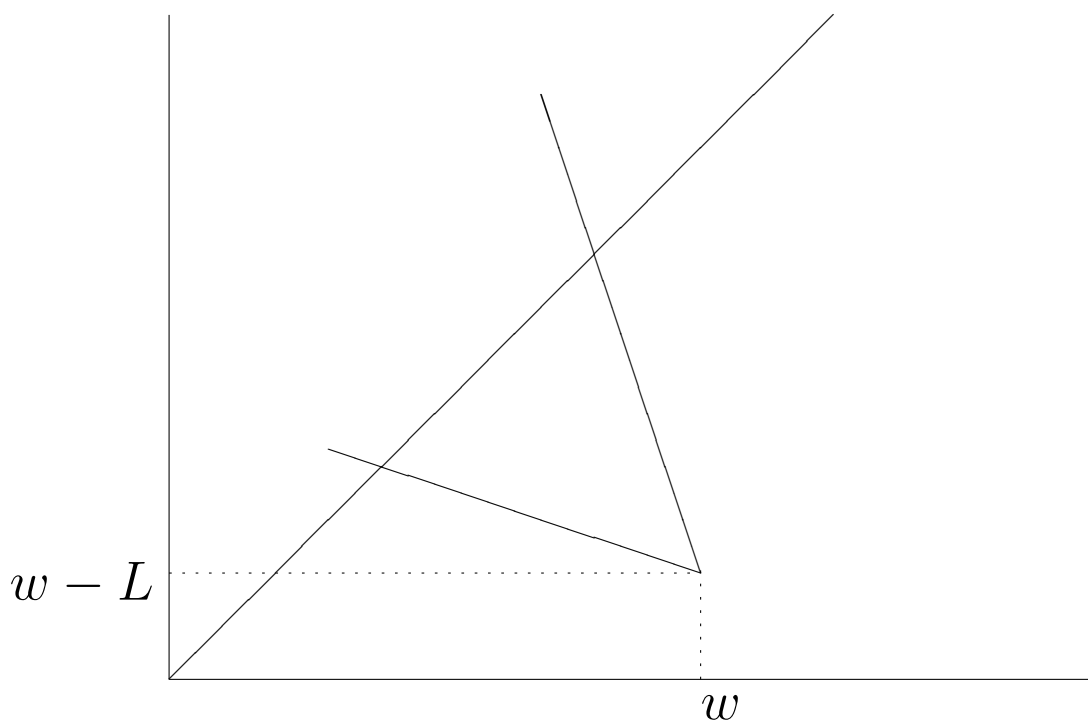
- No equilibrium below the fair pooling line
- For any pooling contract on the fair pooling line “cream skimming” would occur.

There are contracts that would attract only  $l$  types and yield positive expected profits.



## Step 2 – No contracts yielding positive profits

For any contract yielding profits one could find an only slightly better contract that would be preferred by the buyers



## Step 3 – No loss making contracts

As there are no profit yielding contracts cross subsidization is ruled out and loss making contracts are not offered.

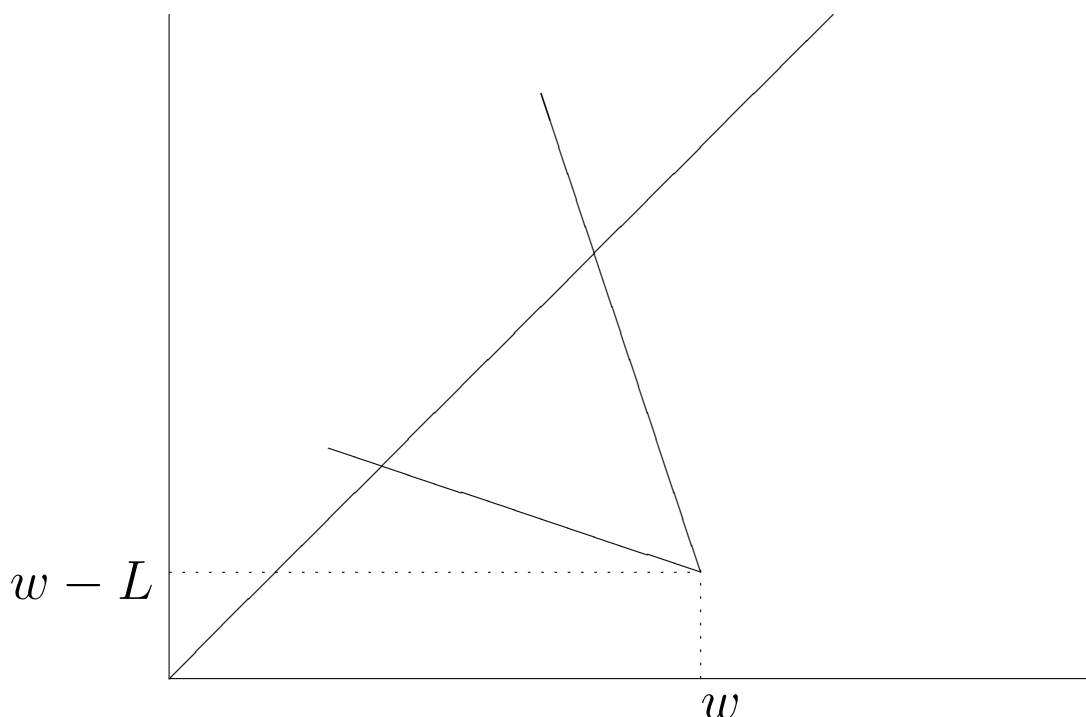
⇒ Equilibrium contracts are going to be on the respective fair insurance lines.

### Step 4 – Full insurance for high risk types

For any contract which is not constituted by the tangential point of the high risk's indifference curve we could find another contract that would yield profits and would be preferred by the high risk type.

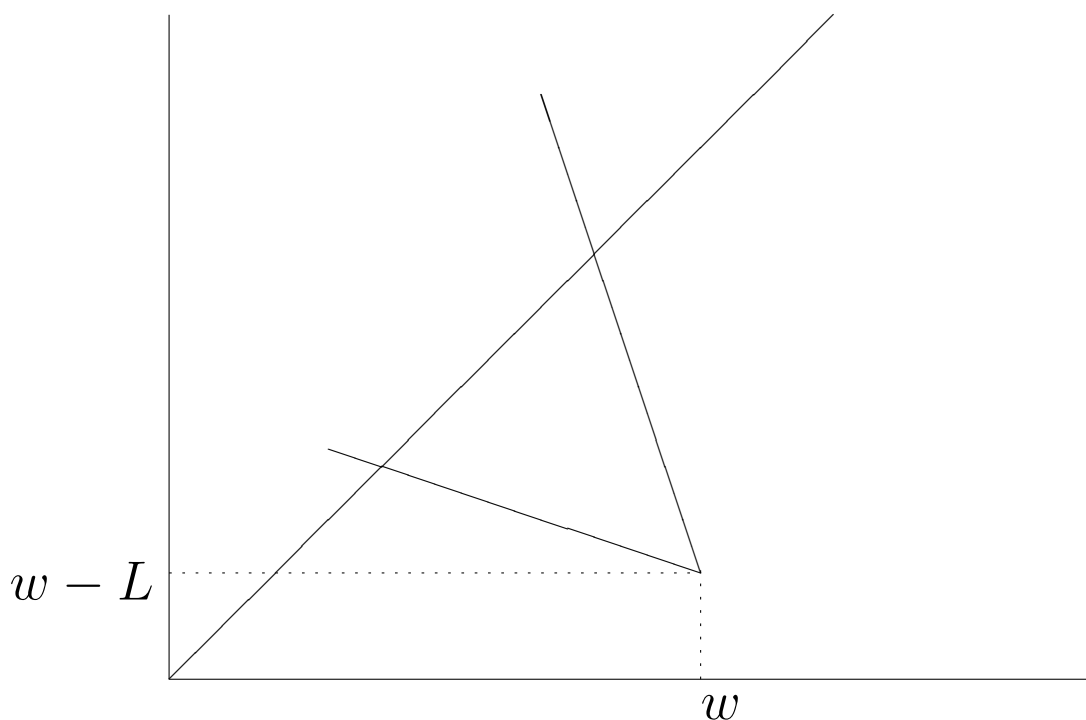
$\Rightarrow$  The contract for the  $h$  types is given by the intersection of the certainty line and the fair insurance line for the  $h$  types.

Note: There is no problem with the low risk types choosing this  $h$  contract.



## Step 5 – Partial insurance for low risk types

Now the contract for the  $l$  types is given by the intersection of the high risk type's indifference curve with the fair insurance line for the  $l$  type. This is the best contract that can be given to the  $l$  type and is not preferred by the  $h$  type.



## The Revelation Principle

For a formal analysis of the model we need an important prerequisite, the so called revelation principle.

(cf. MasColell/Whinston/Green S. 493)

Denote the set of possible states by  $\Theta$ . In searching for an optimal contract, the principal can without loss restrict himself to contracts of the following form:

- After the state  $\theta$  is realized, the agent is required to announce which state has occurred. (e.g. his type)
- The contract specifies an outcome for each possible announcement  $\theta' \in \Theta$ .
- In every state  $\theta \in \Theta$ , the agent finds it optimal to report the state truthfully.

...???

That just says that a principal when looking for an optimal contract can restrict herself – without loss of generality – on the set of direct mechanisms, i.e. those mechanisms that induce truth telling by the agent. She can be sure that there is no non-incentive-compatible contract that would yield a higher expected profit.

**Caveat** There may be problems with the revelation principle when there is scope for renegotiations or when the principal cannot commit to the proposed mechanism.

( cf. Bester and Strausz (2002))

## Formally

From the above stated revelation principle we know that the optimal mechanism will have a set of separating contracts such that each risk type prefers the contract designed for it. As is clear from the above analysis we don't have to worry about the  $l$  type's incentive compatibility constraint. But it is important that the high risk types prefer their contract over the low risk type's contract, i.e.

$$\begin{aligned} & (1 - \pi_h)u(w - \pi_h q_h) + \pi_h u(w - L + (1 - \pi_h)q_h) \\ & \geq \\ & (1 - \pi_h)u(w - \pi_l q_l) + \pi_h u(w - L + (1 - \pi_l)q_l) \end{aligned}$$

has to hold. As the market is competitive I already used that the premium rates are set actuarially fair.

We know that on the interval  $[0, L]$  the left hand side of the inequality is increasing in  $q_h$ .

(1) Set  $q_h = L$  in the above constraint. We can easily see that the inequality can only hold if  $q_l < L$ .

Note that in equilibrium the (weak) inequality will be binding.

(2) If now  $q_h$  were set at any level below  $L$  this would make  $h$  types worse off. In order for the inequality still to hold we would have to reduce  $q_l$  as well. That, in turn, would make  $l$  types worse off, too. So it cannot be optimal.

Thus we get the optimal contracts have  $q_h = L$  and  $q_l^* < L$  satisfying

$$u(w - \pi_h l) = (1 - \pi_h)u(w - \pi_l q_l^*) + \pi_h u(w - L + (1 - \pi_l)q_l^*).$$



## The Rothschild–Stiglitz Result – Summary

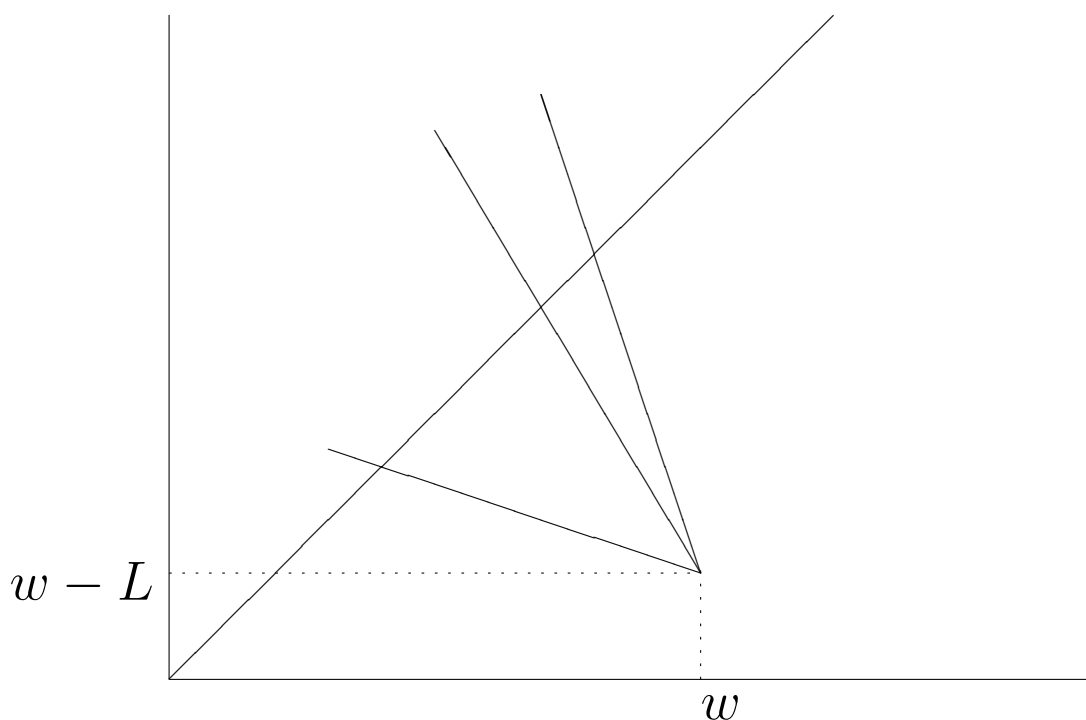
- 1 High risk types get full insurance for their fair premium. (“full cover for high premium”)
- 2 Low risk types get only partial insurance for their fair premium. (“lower premium if only partial cover”)
- 3 Insurance companies break even in expectation.

⇒ There is a market failure as the risk neutral insurer does not take all the risk of the risk averse low risk types. The latter are the only losers from the informational asymmetry.

## Equilibrium non-existence

If the share of  $l$  types,  $\lambda$ , is very high, i.e. the fair pooling line is close to the fair insurance line for the  $l$  types, the RS equilibrium concept runs into a problem.

In such a situation  $l$  and  $h$  types prefer a pooling contract to the menu of separating contracts. But we have shown above that a pooling contract cannot be an equilibrium ...



$\Rightarrow$  There exists no equilibrium

## **What does it mean if there is no equilibrium?**

A model without an equilibrium is like a set of equations without a solution. We feel that the absence of a solution must be due to a faulty specification.

The approach adopted by theorists was to find ways of modifying the equilibrium concept in the RS model in such a way that an equilibrium always exists.

The equilibrium concept in the RS model is that of Nash equilibrium: any one insurer offers contracts (premium and cover) that are a best reaction to those offered by the other sellers, and the RS equilibrium contracts, when they exist, are mutually best replies. Solutions of the non-existence problem have taken the form of extensions to this Nash equilibrium concept.

## Allowing for mixed strategies

**Dasgupta and Maskin (1986)** allow for mixed strategies. In this context, mixed strategies mean that each firm offers different sets of two contracts, each with some probability.

⇒ It always exists an equilibrium.

The exact equilibrium is not known, however we know that

1 firms make zero expected profit

2 with any contract pair offered, the high risks obtain full insurance at a fair or better premium and the low risks obtain partial insurance at an unfair premium.

**But** how should we economically interpret these mixed strategies? Are firms supposed to be randomizing over contracts each year or each day?

In many contexts, mixed strategies are a sensible

concept to use. As a description of the strategic interaction of an insurance markets, however, mixed strategies are more an indication for the limitation of our model.

⇒ We have to look for other answers to the non-existence problem.

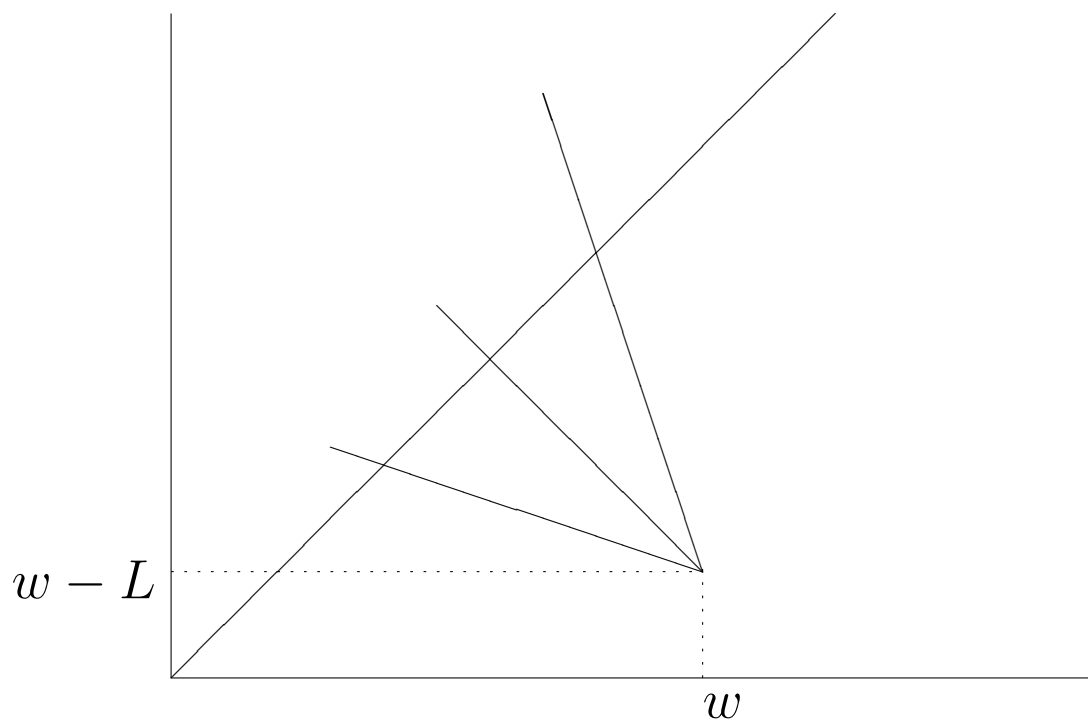
## Wilson's anticipatory equilibrium

The RS equilibrium definition is that there is no menu of contracts outside the equilibrium set that, if offered, makes a profit. In **Wilson's** equilibrium concept (1977), every additional contract should stay profitable even if those contracts which make a loss after the introduction of the new contract, are withdrawn.

Now a pooling contract might survive in equilibrium. Before pooling was unstable because someone could offer a contract only to the low risks. However, in the Wilson concept, if someone tries to attract the low risks only, all others will withdraw their loss making pooling contract, because that contract would be bought by high risks only. Therefore also the high risks choose this newly offered contract. This makes it unattractive to offer in the first place.

This so-called Wilson E2 equilibrium is a partial insurance contract on the fair pooling line where the low risks indifference curve is tangential to that line, i.e.

the best zero-profit pooling contract from the point of view of the low risks.



If there are sufficiently many h types in the population (such that the RS outcome is stable) the Wilson equilibrium coincides with the RS outcome.

## Allowing for cross-subsidization

**Miyazaki (1977)** and **Spence (1978)** allow in addition that firms offer more than one contract. Therefore cross-subsidization between contracts becomes possible. This leads to the so-called WMS equilibrium.

It solves the maximization problem where

- the utility of the low risk type is maximized

under the constraints

- that the high risks will not buy the contract designed for the low risks (incentive constraint)

and

- that the firms make non-negative profit overall.

⇒ The high risks will always obtain full insurance, while the low risks obtain partial insurance.



If  $\lambda$  is sufficiently small, the WMS equilibrium corresponds to the RS outcome. If not, the solution to the above maximization problem is a pair of cross-subsidizing contracts but never a pooling contract. The WMS contracts are second best efficient. There does not exist any other set of contracts which makes no-one worse off and someone better off, given the informational asymmetry.

That is what a competitive market is expected to lead to: Pareto efficient outcomes. This feature of the WMS equilibrium makes it quite popular in the insurance literature.

## Riley's reactive equilibrium

**Riley (1979)** introduced a different equilibrium concept. In his reactive equilibrium, firms shy away from offering deviating contracts if another insurance company would react to such an offer by skimming off the desirable types.

While in the Wilson concept firms anticipate that other firms will withdraw contracts as a result of their entry, here the deviating firms anticipate that at least one other firm will react by offering an additional contract. In that case, the **RS outcome is stable for all values of  $\lambda$** . No one deviates by offering a pooling contract or a pair of cross-subsidizing contracts as in both cases some other firm will profitably 'skim off' the low risk types.

The Riley concept rationalizes the RS outcome even if it does not constitute a Nash equilibrium.

## **Some more attempts**

Hellwig (1987)

Alluding to Wilson (1977) here the firms can in a third stage decide to withdraw some or all of their contracts.

⇒ Wilson Pooling (under some refinements)

Asheim and Nilssen (1996)

Here firms can offer new contracts (only to their own customers). Now they can use cross-subsidizing contracts.

⇒ WMS

Jaynes (1978) and Hellwig (1988)

In stage 1 firms can decide whether there is an exclusivity requirement in the contracts. Later insurance companies can decide whether they exchange information about buyers.

⇒ Wilson Pooling + add-on insurance for high risks (from those firms who do not have exclusivity requirements)

Inderst and Wambach (2001)

Insurance companies have capacity constraints.

⇒ RS

Ania, Troger, and Wambach (2002)

Insurers have imperfect knowledge about buyers utility function etc.. In an evolutionary process they imitate successful firms.

⇒ RS

## A monopoly insurer under adverse selection

So far we analyzed a perfectly competitive insurance market. What are the differences if there is a sole monopoly insurer supplying insurance cover?

Consider a situation where we have a continuum of buyers with mass 1. The insurer can set premium  $P_i$  and the amount of cover  $\phi_i$ . Thus the insurer's problem takes the following form:

$$\max_{P_l, P_h, \phi_l, \phi_h} \Pi = \lambda(P_l - \pi_l \phi_l) + (1 - \lambda)(P_h - \pi_h \phi_h)$$

s.t.

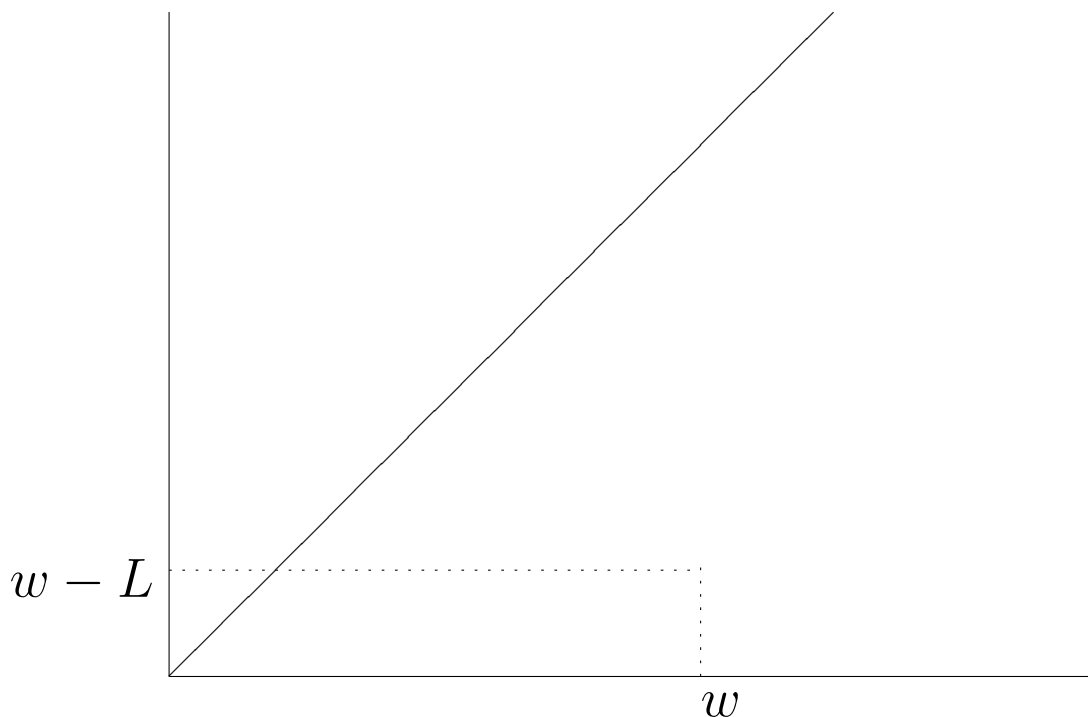
$$\text{PC (h)} \quad EU_h(P_h, \phi_h) \geq \bar{U}_h$$

$$\text{PC (l)} \quad EU_l(P_l, \phi_l) \geq \bar{U}_l$$

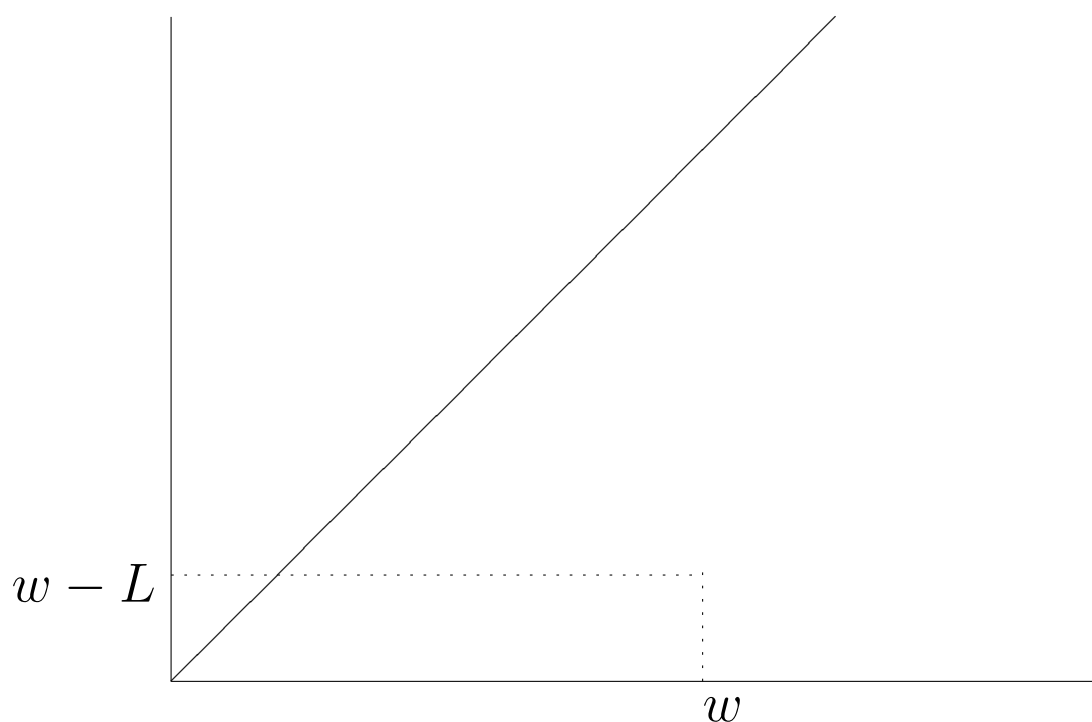
$$\text{IC (h)} \quad EU_h(P_h, \phi_h) \geq EU_h(P_l, \phi_l)$$

$$\text{IC (l)} \quad EU_l(P_l, \phi_l) \geq EU_l(P_h, \phi_h).$$

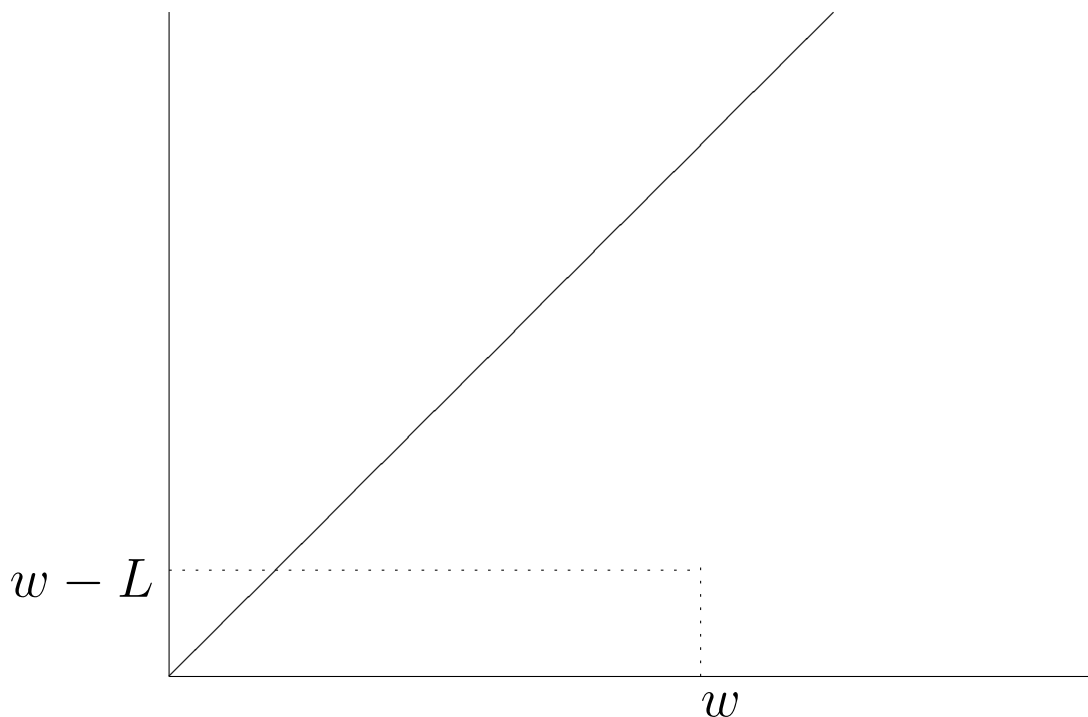
**PC (h)**, the participation constraint for the high risk types, is not binding. We can see that easily from the figure, because for any contract where income is shifted into the “loss” state the low risk type’s outside option indifference curve lies above the  $h$  type’s outside option indifference curve. Any contract for the  $l$  types gives the  $h$  types a rent.



**PC (1)**, the participation constraint for the low risk types, is binding. If it were not binding we could always find contracts that would be acceptable for  $h$  and  $l$  types and would yield higher profits.



**IC (1)**, the incentive constraint for the low risk types, is not binding. We know that the  $l$  types do not get a rent. So if IC (1) were binding the contract for the  $h$  types were on the  $l$  type's outside option indifference curve. We can easily find contracts for the  $h$  types that yield higher profit while not violating any constraint.





**IC (h)**, the incentive constraint for the high risk types, is binding. If it were not the problem would coincide with the one under symmetric information. There a menu of contracts not satisfying IC (h) is optimal. Thus IC (h) has to be binding.

Now the problem is reduced to a simple Lagrange problem:

$$\max_{P_l, P_h, \phi_l, \phi_h} \Pi = \lambda(P_l - \pi_l \phi_l) + (1 - \lambda)(P_h - \pi_h \phi_h)$$

s.t.

$$\text{PC (l)} \quad EU_l(P_l, \phi_l) = \bar{U}_l$$

$$\text{IC (h)} \quad EU_h(P_h, \phi_h) = EU_h(P_l, \phi_l).$$

From the first order conditions with respect to  $P_h$  and  $\phi_h$  one can see that marginal utility, and thus final wealth, for the h types are equal in both states of the world, i.e.  $h$  types get fully insured. (check that)

From the first order conditions with respect to  $P_l$  and  $\phi_l$  we can see that for the low risk types marginal utility in the “loss” state is higher, i.e. their final wealth in this state is lower. Thus low risks receive only partial insurance. (check that)

## Monopoly under Adverse Selection – Summary

(1) Pooling is never optimal.

(2) High risks receive a rent and are fully insured.

(3) Low risks receive no rent and are only partially insured. The level of partial insurance depends on the share of high risks in the population. Note that the monopolist has to leave a rent to the  $h$  types in order to separate the types. Now if there are only few  $l$  types in the population the monopolist will forego any rents from the  $l$  types but extract all the rent from the  $h$  types. They then get the full insurance contract where their outside option indifference curve is tangential to their fair insurance line.

**Note:** If the insurer has additional instruments/information to discriminate between  $h$  and  $l$  types she will use them. We will cover the issues of categorical discrimination (problem 6–2) and endogenous discrimination (problem 6–3) in class.

## Longterm contracts – Basic idea

Now we consider a longer time horizon. The loss probabilities are to be interpreted as per period loss probabilities. As the risk type of an insuree is exogenously given we will learn over time his true risk type.

So the question arises whether the insurer can do better by writing longterm/multi-period contracts. Now she can condition the contract (premium and cover) on the previous track record of the insuree.

### Examples:

- Unemployment insurance: The payment decreases in the duration of unemployment.
- Car insurance: Premium depends on the number of previous accidents. (experience rating, bonus–malus–system)

## Longterm contracts – Basic structure

For a start consider the following simple model:

- 2 periods
- same initial income in periods 1 and 2; no savings
- premium in period 2 ( $P^2$ ) conditional on loss in period 1
- cover in period 2 ( $\phi^2$ ) conditional on loss in period 1

Contract for  $h$  types:

$$P_h^1 = P_h^2(Loss) = P_h^2(NoLoss)$$

and

$$\phi_h^1 = \phi_h^2(Loss) = \phi_h^2(NoLoss)$$

$\Rightarrow$  High risks are fully insured. The longterm contract is just a replication of two short term contracts.

Contract for  $l$  types:

$$P_l^2(NoLoss) < P_l^1 < P_l^2(Loss)$$

and

$$\phi_l^2(NoLoss) > \phi_l^1 > \phi_l^2(Loss)$$

$\Rightarrow$  Low risks are not fully insured. They face a risk over time and are rewarded if there was no loss but punished if there was a loss. The  $h$  types for whom this risk is higher will not choose the low risk type's contract.

### More than 2 periods

$P_l^T$  increases in the number of losses

$\phi_l^T$  decreases in the number of losses

For  $T \rightarrow \infty$  we converge to the FB solution as the per period “punishment” can be arbitrarily small.

**Note:** It is important that there is no saving. If the insurees could insure themselves via unobservable savings the problem is more subtle.

## Renegotiation

Idea: Over time the insurer learns about the insured's true type. This information could be used to design a more efficient contract (for the  $l$  types).

Or: Longterm contracts are prohibited by law.

### Renegotiation before contract starts

By choosing the respective separating contracts we know the buyers types for sure. So we could do better and offer the  $l$  type, directly after the initial  $l$  contract is signed, a full insurance contract for the fair  $l$  premium.

What would happen? The  $h$  types would anticipate this and would pick the  $l$  contract in the first place.

⇒ Problem ...

### Renegotiation later on

From the observation in period 1 the insurer receives additional information on the true risk type of a buyer. Now she can offer a better contract for period 2. An interesting question is whether to make profits in the beginning and losses later on (theoretical suggestion) or vice versa (empirically backed suggestion, “low-balling”). Note that in equilibrium renegotiation will not occur. But the mere possibility changes the nature of the problem.