# Insurance Markets - Demand for Insurance
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# 2 Basic Models

We can think of 2 ways to model insurance demand:

(i) *The $q$– Model* A certain payment, the **premium**, is exchanged for the promise to pay partial or full compensation (**cover**) for **loss** resulting from carefully specified **loss events**. So premium is exchanged for cover.

(ii) *The $y$– Model* As state-contingent incomes. Income is reduced in states of the world in which the specified events do not happen, and increased in states in which the events do happen, as compared to the situation without insurance.

The two models are fully equivalent. The $q$-model is often easier to handle mathematically. The advantage of the $y$-model is that it allows the similarities with the standard consumer theory to be exploited.

Simplest possible model: 2 states 1 and 2.

- income in the no-loss state: $y_1 = y$
- income in the loss state: $y_2 = y - L$
- probability of loss $L$ is $\pi$
- expected value of income without insurance:

$$\bar{y} = (1-\pi)y + \pi(y - L) = y - \pi L$$

- expected value of income loss: $\pi L$
- expected utility in the absence of insurance:

$$\bar{u}^0 = (1-\pi)u(y) + \pi u(y - L)$$

In the absence of insurance the individual has an uncertain income endowment.

## The $q$–Model

insurer offers cover $q$ at a *premium rate* $p$, where $p$ is a pure number (as is a probability). The *premium amount* $P$ (Euro) is $pq$.

Note that assuming the marginal price of cover $p$ to be constant is restrictive in the sense that it rules out convex price schemes.

the buyer chooses $q \geq 0$ to maximize

$$\bar{u}(q) = (1-\pi)u(y-pq) + \pi u(y-L+(1-p)q)$$

standard first order (Kuhn-Tucker) condition

$$\begin{aligned} \bar{u}_q &= -p(1-\pi)u^{'}(y-pq^*) + (1-p)\pi u^{'}(y-L+(1-p)q^*) \leq 0 \\ q^* &\geq 0 \\ \bar{u}_q q^* &= 0 \end{aligned}$$

Note that the second order condition is, as required for an optimum, globally negative.

Then

$$q^* > 0 \Rightarrow \frac{1-p}{p} = \frac{(1-\pi)u^{'}(y-pq^*)}{\pi u^{'}(y-L+(1-p)q^*)}$$

while

$$\frac{1-p}{p} < \frac{(1-\pi)u^{'}(y-pq^*)}{\pi u'(y-L+(1-p)q^*)} \Rightarrow q^* = 0$$

This is equivalent to saying that the demand for cover will be only positive if the price $p$ is "not too high", namely

$$p < \frac{\pi u^{'}(y-L+(1-p)q^*)}{(1-\pi)u^{'}(y-pq^*) + \pi u'(y-L+(1-p)q^*)}.$$

From now on we will generally neglect the Kuhn–Tucker conditions, implicitly focussing on the economically more interesting case of an interior solution.

Assuming $q^* > 0$ and rearranging the first order condition

$$\bar{u}_q = -p(1-\pi)u^{'}(y - pq^*) + (1-p)\pi u^{'}(y - L + (1-p)q^*) \leq 0 q^* = 0$$

the following must hold:

- $p = \pi \Leftrightarrow q^* = L$
  with a *fair premium* there is *full cover*
- $p > \pi \Leftrightarrow q^* < L$
  with a *positive loading* there is *partial cover*
- $p < \pi \Leftrightarrow q^* > L$
  with a *negative loading* there is *more than full cover.*

For example

$$p = \pi \Leftrightarrow u^{'}(y - pq^{*}) = u^{'}(y - L + (1-p)q^{*}) \Leftrightarrow q^{*} = L$$

## Comparative Statics

explore the relationships between the optimal value of the endogenous variable, the demand for insurance, and the exogenous variables that determine it, $p, \pi, L, y$.

Note: if we assume the premium is always fair, full cover is always bought $\Rightarrow$ comparative statics analysis is trivial.

assume (realistically) that $p > \pi$ and thus $0 < q^* < L$

first order condition is

$$\bar{u}_q = -p(1-\pi)u^{'}(y - pq^*) + (1-p)\pi u^{'}(y - L + (1-p)q^*) = 0$$

Applying standard methods of comparative statics (implicit function theorem) we have that

$$(1) \quad \frac{\partial q^*}{\partial y} = -\frac{\bar{u}_{qy}}{\bar{u}_{qq}}$$

$$(2) \quad \frac{\partial q^*}{\partial L} = -\frac{\bar{u}_{qL}}{\bar{u}_{qq}}$$

$$(3) \quad \frac{\partial q^*}{\partial p} = -\frac{\bar{u}_{qp}}{\bar{u}_{qq}}$$

$$(4) \quad \frac{\partial q^*}{\partial \pi} = -\frac{\bar{u}_{q\pi}}{\bar{u}_{qq}}$$

because of risk aversion ($u^{''} < 0$) it is easy to show that $\bar{u}_{qq} < 0$

$\Rightarrow$ sign of derivatives is determined by that of the numerator

Thus we have for (1)

$$\bar{u}_{qy} = -p(1-\pi)u''(y-pq^*) + (1-p)\pi u''(y-L+(1-p)q^*) \leq / \geq 0$$

indeterminacy of the sign of this effect is no surprise

as in standard consumer theory income effects can go either way and insurance cover can be an inferior or a normal good

relate this to the buyer's risk preferences:

$$y_1^* \equiv y - pq^* \text{and} y_2^* \equiv y - L + (1-p)q^*$$

are the optimal incomes in the two states

$y_1^* > y_2^*$ because of partial cover rearranging the first order condition gives

$$p(1-\pi) = \frac{(1-p)\pi u^{'}(y_2^*)}{u^{'}(y_1^*)}$$

Substituting gives

$$\bar{u}_{qy} = -u^{''}(y_1^*)\frac{(1-p)\pi u^{'}(y_2^*)}{u^{'}(y_1^*)} + (1-p)\pi u^{''}(y_2^*)$$

$$\bar{u}_{qy} = (1-p)\pi u^{'}(y_2^*)[\frac{u^{''}(y_2^*)}{u^{'}(y_2^*)} - \frac{u^{''}(y_1^*)}{u^{'}(y_1^*)}]$$

Recall now the Arrow-Pratt measure of (absolute) risk aversion

$$A(y) \equiv -\frac{u^{''}(y)}{u^{'}(y)}$$

We can then write

$$\bar{u}_{qy} = (1-p)\pi u^{'}(y_2^*)[A(y_1^*) - A(y_2^*)]$$

Thus

$$\bar{u}_{qy} \geq / \leq 0 \quad \Leftrightarrow \quad A(y_1^*) \geq / \leq A(y_2^*)$$

insurance cover is a normal good if risk aversion increases with income (IARA, $A(y_1^*) > A(y_2^*$ with $y_1^* > y_2^*$)

insurance cover is an inferior good if risk aversion decreases (DARA) or is constant with income (CARA, $A(y_1^*) \leq A(y_2^*)$)

The intuition is straightforward: if an increase in income increases one's willingness to bear risk, then one's demand for insurance falls.

Next we analyze the effect of an **increase in the loss** $L$

$$\bar{u}_{qL} = -(1-p)\pi u^{''}(y_2^*) > 0$$

Thus, as we would intuitively expect, an increase in loss increases the demand for cover, *other things being equal.*

Thirdly we analyze the effect of an **increase of the premium rate**

$\bar{u}_{qp} = -[(1-\pi)u^{'}(y_1^*) + \pi u^{'}(y_2^*)] + [p(1-\pi)u^{''}(y_1^*) - (1-p)\pi u^{''}(y_2^*)]q^*$

But notice that the second term is just $-u_{qy}q^*$. In fact we have a standard Slutsky equation, which we can write as

$$\frac{\partial q^*}{\partial p} = -\frac{\bar{u}_{qp}}{\bar{u}_{qq}} = \frac{(1-\pi)u^{'}(y_1^*) + \pi u^{'}(y_2^*)}{\bar{u}_{qq}} + q^* \frac{u_{qy}}{\bar{u}_{qq}}$$

The first term is the substitution effect, and is certainly negative ($\bar{u}_{qq} < 0$).

The second term is the income effect and, as we have seen, could be positive or negative (or zero).

If insurance is an inferior good this income effect is negative and so the demand for cover certainly falls as the premium rate (price) rises. That is, there is no ambiguity if absolute risk aversion increases (or is constant) with income.

If insurance is a normal good the income effect is positive and so works against the substitution effect. That is, insurance may be a Giffen good if risk aversion decreases sufficiently with income.

The intuition is easy to see. A fall in the premium rate reduces the price of income in state 2 relative to that in state 1, and so, with utility held constant, $y_2$ will be substituted for $y_1$, implying an increased demand for cover. However, the fall in premium also increases real income, to an extent dependent on the amount of cover already bought, $q^*$, and this will tend to reduce the demand for insurance if risk aversion falls with income, and increase it if risk aversion increases with income.

Finally we look at the effect of an **increase in the loss probability**.

$$\bar{u}_{q\pi} = pu^{'}(y_1^*) + (1-p)u^{'}(y_2^*) > 0$$

Thus, as we would expect, an increase in the risk of loss increases demand for cover.

**Note**, however, there is a strong "other things equal" assumption here. In general we would not expect the premium to remain constant when the loss probability changes, though we need some theory of the supply side of the market before we can predict how it would change. Thus the above does not give the full market comparative statics of a change in loss probability. Exactly the same point applies to the change in $L$.