6. Pareto efficient risk allocation

Think of an economy consisting of two risk averse individuals, a and b. They face two possible states of the world. The probability for the realization of state 1 is $1 - \pi$, the probability for state 2 is π. In state 1, a's (b's) wealth is w_{1a} (w_{1b}), in state 2 it is w_{2a} (w_{2b}). The society's wealth in the two states of the world is simply defined as the sum of the individuals' wealth ($w_1 = w_{1a} + w_{1b}$ and $w_2 = w_{2a} + w_{2b}$). The two agents can now write a contract, in which they reallocate their wealth in both of the two possible states of the world. Draw the situation in an Edgeworth-box and show the set of Pareto-efficient risk allocations in case where there is

a) no social risk (i.e., social wealth is the same in both states of the world: $w_{1a} + w_{1b} = w_{2a} + w_{2b}$);

b) social risk (i.e., social wealth differs in the two states of the world, respectively);

c) social risk and one of the agents is risk neutral.

7. Risk allocation, financial markets

Consider the following situation: There are two states of the world $s \in \{1, 2\}$ that occur with π and $1 - \pi$, respectively. Two individuals, a and b, with endowments $e_{1a},\ e_{2a},\ e_{1b},\ e_{2b}$ can trade income y_{si} between the states. Let income in state one be the numeraire; income in state two can be traded at price p.

a) First, take the general welfare maximization problem with $e_{1a} = 1,\ e_{2a} = 0,\ e_{1b} = 0,\ e_{2b} = 1$ and determine a Pareto-efficient outcome.

b) Take the following numerical example and derive the market outcome: $u_i(y_{1i}, y_{2i}) = \pi ln y_{1i} + (1 - \pi) ln y_{2i}\ for\ i \in \{a, b\}$ and $e_{1a} = 1,\ e_{2a} = 0,\ e_{1b} = 0,\ e_{2b} = 1$

c) Take the following numerical example and derive the market outcome: $u_i(y_{1i}, y_{2i}) = \pi ln y_{1i} + (1 - \pi) ln y_{2i}\ for\ i \in \{a, b\}$ and $e_{1a} = 1,\ e_{2a} = 0,\ e_{1b} = 0,\ e_{2b} = 2$

d) Most of the time, agents have different beliefs about π. Assume $\pi_a = \frac{1}{3}$ and $\pi_b = \frac{2}{3}$ in the setting of c). Derive the market outcome.

e) Does Gossen's second law hold in b), c) and d)?

f) Would the individuals appreciate it if someone told them the true state of the world before the transaction takes place?

g) Instead of writing a contract the individuals now have access to the financial markets and trade two assets, so called Arrow-securities, that yield one unit of income in exactly one state of the world and zero in all the other states. Let the price of the first asset be $p_1 = 1$. Find the optimal amounts q_{1i} and q_{2i} that will be traded and calculate p_2. In the presence of financial markets, why can there still be scope for insurance markets?

8. Informal insurance agreements

Suppose that two individuals, a and b, agree on risk sharing, according to exercises **6.** and **7.**, but the transfer payment is not contractible. In the absence of binding contracts they might design a self enforcing mechanism if the game is repeated. Take the setting in the diagram as given. Let $2\pi_e$ and $2\pi_d$ denote the probabilities for equal and different incomes, respectively. To start with, assume that $\pi_e = 0$. Let $u(y_i) = \sqrt{y_i}$ be a person's utility derived from his income level.

	$y_b = 0$	$y_b = 1$	
$y_a = 0$	π_e	π_d	0.5
$y_a = 1$	π_d	π_e	0.5
	0.5	0.5	1

Derive:

a) the first best transfer;

b) the expected utility per person and period without insurance;

c) the expected utility per person and period with transfer $\theta = 0.36$;

d) the maximum value for the discount rate r, so that the informal insurance contract with a second best transfer of $\theta = 0.36$ is still implementable.

Assume now the more general case $\pi_e > 0$.

e) Calculate the correlation coefficient $\rho(y_a, y_b)$. What has to hold for r and ρ so that the informal contract with $\theta = 0.36$ is self enforcing?

9. Economies of scale

It is often stated that there are economies of scale in the insurance industry.

a) Think about a way to model an insurance company's cost function. Especially try to find a way to plausibly describe an insurance company's marginal costs.

b) Now assume a standard risk neutral utility function of the form $u(w) = aw$. Is it, for this specific utility function, true that there are economies of scale in the insurance business?
(Economies of scale means that average costs are decreasing.)

c) Do you think there are economies of scale in the real world insurance industry? (Hint: Think of the firms' attitude towards risk and other than risk costs.)

10. The Arrow-Lind Theorem

The compensating risk premium k is defined as $u(y) = \mathrm{E}u[y + k + x]$ According to the Arrow-Lind Theorem both the premium of each risk avers insurer and the sum of all premia decrease as the number of insurers increases.

a) Give an intuition for the compensating risk premium, e.g. compare the compensating risk premium with the standard equivalent risk premium.

b) Suppose that $\mathrm{E}x = 0$ and that the risky project is shared among all insurers. Using a Taylor-approximation find an approximative value of k.

c) Then show that both the individual premium and the sum of all premia decrease as the number of investors increases.