Seminar für Versicherungswissenschaft
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Diplomprüfung für Volkswirte
Allocation and Environment

You have 120 minutes to write the exam. There are: 10 points for question 1 and 30 points each for questions 2-5. You must answer question 1, and three of the questions 2-5 (if you answer more than 3 of these, the answer that gets the highest marks will not count).

All answers must be justified!

Good luck!

1. What do the two fundamental theorems of welfare economics tell us? Discuss them in relation to the issues of equity and efficiency. Discuss the problem of their applicability in reality.

2. A car-owner wants to sell his car to a car dealer. The car can be of good or bad quality. The seller's reservation price for a good car is 1000, for a bad car 200. The dealer's willingness to pay is 1200 for a good car and 500 for a bad car.

   (a) Assume that both the seller and the dealer know the quality of the car. Will the car be sold in the equilibrium? If yes, at what price? Is the result efficient (explain your answer)? What is the result if neither seller nor dealer know the quality of the car, but both are risk neutral and know that the probability that the car is good is 0.5.

   (b) Now assume that only the dealer knows the quality of the car, the seller knows that he knows and also knows the dealer's willingness to pay for a good and bad car. The seller is risk neutral and knows that the probability that he has a good car is 0.5. Can a sale take place under these conditions? If yes, what does the occurence of a sale depend on? Is the outcome efficient?

3. The demand for crossings of a bridge is given by the function $p(x) = a - bx$, with $a, b > 0$. The capacity of the bridge is limited, so that above a particular level congestion sets in.

   (a) What condition holds for the equilibrium traffic flow across the bridge? Why is this not efficient? What is the cause of the inefficient outcome in this model? Illustrate your answer with an appropriate diagram.

   (b) Assume that the government wants to maximise social net benefits by setting a toll. Formulate and solve the corresponding optimisation problem. Show in the figure given in (a) the optimal toll and the achieved gain in welfare. Explain the effects of the toll in this model.

   (c) Show in a separate diagram the gains and losses in benefits and costs created by the toll, as well as the toll revenue, and explain these. Show that it is in priciple possible to make everyone better off with the toll. What practical problems do you see in this? What are the arguments in favour of a new bridge? Under what circumstances would this be efficient even without a toll? Under what conditions is the construction of a new bridge justified?

4. The demand for a non-renewable resource is given by the function $p_t(q_t) = a - bq_t, \quad t = 0, \ldots, T, \quad a, b > 0$. Extraction of the resource is subject to constant marginal cost $c > 0$.

   (a) Assume that there is an infinite amount of the resource available. Derive algebraically the conditions that determine the optimal extraction rate q^* in each period that maximises net social benefits.

   (b) Assume in what follows that $T = 1$ (2-period model) and only $R < (T + 1)q^*$ of the resource is available. We wish to maximise the present value of net social benefit (future benefits are discounted by the factor $1/(1+r)$, where r is the interest rate). Formulate the optimisation problem and derive the first order conditions.

(c) Show algebraically that Hotelling's Rule must be satisfied at the optimum. What is the economic meaning of the terms $\mu_t = p_t - c$ and the Lagrange multiplier in this model? Give a diagram for a 1- and 2-period model respectively which make clear the relevant magnitudes.

(d) In $t = 1$ there is a surprise discovery of an amount $\tilde{R}$ of the resource. What is the effect of this on the optimal time path of μ_t? Explain why it would have been better if $\tilde{R}$ had instead been discovered at $t = 0$.

5. Two firms pollute two cities. The benefit functions of the firms are given by:

$$\begin{aligned} B_1(e_1) &= 10e_1 - e_1^2/2 \\ B_2(e_2) &= 4e_2 - e_2^2/2 \end{aligned}$$

The pollution matrix is

$$\mathbf{A} = \begin{pmatrix} 4 & 1 \\ 1 & 2 \end{pmatrix}$$

The planner wants to ensure that pollution in the two cities is no greater than $\bar{q}_1 = 10$ units in city 1 and $\bar{q}_2 = 9$ units in city 2.

(a) Find the prices, demands for pollution licenses and emission levels at an equilibrium in the markets for pollution licenses. Which constraints are binding in equilibium?

(b) Illustrate the nature of the solution to a „fictitious planner's“ problem, which seeks to maximise total benefits subject to pollution constraints.

(c) Give the reason why the answer to (a) can be used to find the answer to (b).

(d) Suppose that instead of a market in pollution licenses, a market in emission licenses is set up. Explain why this is not equivalent to a market in pollution licenses. Explain how it may be made equivalent to the market in pollution licenses.

(e) Briefly discuss the arguments for and against introducing markets in pollution licenses in practice.